CURVATURE PROPERTIES OF TWO NAVEIRA CLASSES OF RIEMANNIAN PRODUCT MANIFOLDS

Dobrinka Gribacheva

Abstract. The main aim of the present work is to obtain some curvature properties of the manifolds from two classes of Riemannian product manifolds. These classes are two basic classes from Naveira classification of Riemannian almost product manifolds.

Key words: Riemannian almost product manifold, Riemannian metric, almost product structure, curvature tensor.

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Introduction

A Riemannian almost product manifold (M, P, g) is a differentiable manifold M with an almost product structure P and a Riemannian metric g such that $P^2 x = x$ and $g(Px, Py) = g(x, y)$ for any tangent vectors x and y.

K. Yano initiated in [7] the study of Riemannian almost product manifolds. A. M. Naveira gave in [4] a classification of these manifolds with respect to the covariant derivative ∇P, where ∇ is the Levi-Civita connection of g. This classification is very similar to the Gray-Hervella classification in [1] of almost Hermitian manifolds. In [6] M. Staikova and K. Gribachev have obtained a classification of the Riemannian almost product manifolds for which $\mathrm{tr} P = 0$. In this case the manifold is even-dimensional. The basic class $\mathcal{W}_1$ from the Staikova-Gribachev classification is the class of conformal Riemannian P-manifolds or shortly $\mathcal{W}_1$-manifolds. It is an analogue of the class of conformal Kähler manifolds in almost Hermitian geometry. It is valid $\mathcal{W}_1 = \overline{\mathcal{W}}_3 \oplus \overline{\mathcal{W}}_6$, where $\overline{\mathcal{W}}_3$ and $\overline{\mathcal{W}}_6$ are basic classes from the Naveira classification. In some sense these manifolds have dual geometries.

The main aim of the present work is to obtain some curvature properties of the manifolds from the Naveira classes $\overline{\mathcal{W}}_3$ and $\overline{\mathcal{W}}_6$ with respect to ∇.

The paper is organized as follows. In Sec. 1 we give necessary facts about Riemannian almost product manifolds, the classes $\mathcal{W}_1$, $\overline{\mathcal{W}}_3$, $\overline{\mathcal{W}}_6$ and the notion a Riemannian P-tensor on a Riemannian almost product manifold, which is an analogue of the notion of a Kähler tensor in Hermitian geometry. In Sec. 2 we obtain curvature properties of the $\overline{\mathcal{W}}_3$-manifolds with respect to ∇. Using the same methods as in Sec. 2, we obtain in Sec. 3 for the analogous curvature properties of the $\overline{\mathcal{W}}_6$-manifolds same or similar algebraic expressions.

1. Preliminaries

Let (M, P, g) be a *Riemannian almost product manifold*, i.e. a differentiable manifold M with a tensor field P of type $(1, 1)$ and a Riemannian metric g such that $P^2 x = x$, $g(Px, Py) = g(x, y)$ for any x, y of the algebra $\mathfrak{X}(M)$ of the smooth vector fields on M. Further x, y, z, u, w will stand for arbitrary elements of $\mathfrak{X}(M)$ or vectors in the tangent space $T_c M$ at $c \in M$.

In this work we consider manifolds (M, P, g) with $\mathrm{tr} P = 0$. In this case M is an even-dimensional manifold. We assume that $\dim M = 2n$.

In [4] A. M. Naveira gives a classification of Riemannian almost product manifolds with respect to the tensor F of type (0,3), defined by

(1.1) $$F(x, y, z) = g\left((\nabla_x P) y, z\right),$$

where ∇ is the Levi-Civita connection of g. The tensor F has the properties:

$$F(x, y, z) = F(x, z, y) = -F(x, Py, Pz), \quad F(x, y, Pz) = -F(x, Py, z).$$

Using the Naveira classification, in [6] M. Staikova and K. Gribachev give a classification of Riemannian almost product manifolds (M, P, g) with $\mathrm{tr} P = 0$. The basic classes of this classification are $\mathcal{W}_1$, $\mathcal{W}_2$ and $\mathcal{W}_3$. Their intersection is the class $\mathcal{W}_0$ of the *Riemannian P-manifolds* ([5]), determined by the condition $F = 0$. This class is an analogue of the class of Kähler manifolds in the geometry of almost Hermitian manifolds.

The class $\mathcal{W}_1$ from the Staikova-Gribachev classification contains the manifolds which are locally conformal equivalent to Riemannian P-manifolds. This class plays a similar role of the role of the class of the conformal Kähler manifolds in almost Hermitian geometry. We will say that a manifold from the class $\mathcal{W}_1$ is a $\mathcal{W}_1$-*manifold*.

The characteristic condition for the class $\mathcal{W}_1$ is the following

$$\mathcal{W}_1 : F(x,y,z) = \tfrac{1}{2n}\big\{g(x,y)\theta(z) - g(x,Py)\theta(Pz)$$
$$+ g(x,z)\theta(y) - g(x,Pz)\theta(Py)\big\},$$

where the associated 1-form θ is determined by $\theta(x) = g^{ij}F(e_i,e_j,x)$. Here g^{ij} will stand for the components of the inverse matrix of g with respect to a basis $\{e_i\}$ of T_cM at $c \in M$. The 1-form θ is *closed*, i.e. $\mathrm{d}\theta = 0$, if and only if $(\nabla_x\theta)\,y = (\nabla_y\theta)\,x$. Moreover, $\theta \circ P$ is a closed 1-form if and only if $(\nabla_x\theta)\,Py = (\nabla_y\theta)\,Px$.

In [6] it is proved that $\mathcal{W}_1 = \overline{\mathcal{W}}_3 \oplus \overline{\mathcal{W}}_6$, where $\overline{\mathcal{W}}_3$ and $\overline{\mathcal{W}}_6$ are the classes from the Naveira classification determined by the following conditions:

$$\overline{\mathcal{W}}_3 : \quad F(A,B,\xi) = \tfrac{1}{n}\,g(A,B)\theta^v(\xi), \quad F(\xi,\eta,A) = 0,$$
$$\overline{\mathcal{W}}_6 : \quad F(\xi,\eta,A) = \tfrac{1}{n}\,g(\xi,\eta)\theta^h(A), \quad F(A,B,\xi) = 0,$$

where $A,\ B,\ \xi,\ \eta \in \mathfrak{X}(M)$, $PA = A$, $PB = B$, $P\xi = -\xi$, $P\eta = -\eta$, $\theta^v(x) = \tfrac{1}{2}\left(\theta(x) - \theta(Px)\right)$, $\theta^h(x) = \tfrac{1}{2}\left(\theta(x) + \theta(Px)\right)$. In the case when $\mathrm{tr}P = 0$, the above conditions for $\overline{\mathcal{W}}_3$ and $\overline{\mathcal{W}}_6$ can be written for any $x,\ y,\ z$ in the following form:

$$(1.2) \qquad \overline{\mathcal{W}}_3 : \quad F(x,y,z) = \tfrac{1}{2n}\big\{[g(x,y) + g(x,Py)]\,\theta(z)$$
$$+ [g(x,z) + g(x,Pz)]\,\theta(y)\big\}, \quad \theta(Px) = -\theta(x),$$

$$(1.3) \qquad \overline{\mathcal{W}}_6 : \quad F(x,y,z) = \tfrac{1}{2n}\big\{[g(x,y) - g(x,Py)]\,\theta(z)$$
$$+ [g(x,z) - g(x,Pz)]\,\theta(y)\big\}, \quad \theta(Px) = \theta(x).$$

We will say that a manifold from the class $\overline{\mathcal{W}}_3$ (resp., $\overline{\mathcal{W}}_6$) is a $\overline{\mathcal{W}}_3$-*manifold* (resp., $\overline{\mathcal{W}}_6$-*manifold*).

In [6], a tensor L of type (0,4) with properties

$$(1.4) \qquad L(x,y,z,w) = -L(y,x,z,w) = -L(x,y,w,z),$$

$$(1.5) \qquad L(x,y,z,w) + L(y,z,x,w) + L(z,x,y,w) = 0$$

is called a *curvature-like tensor*. Such a tensor on a Riemannian almost product manifold (M,P,g) with the property

$$(1.6) \qquad L(x,y,Pz,Pw) = L(x,y,z,w)$$

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is called a *Riemannian P-tensor* in [3]. This notion is an analogue of the notion of a Kähler tensor in Hermitian geometry.

Let S be a (0,2)-tensor on a Riemannian almost product manifold. In [6] it is proved that

$$\psi_1(S)(x, y, z, w) = g(y, z)S(x, w) - g(x, z)S(y, w)$$
$$+ S(y, z)g(x, w) - S(x, z)g(y, w)$$

is a curvature-like tensor if and only if $S(x, y) = S(y, x)$, and the tensor

$$\psi_2(S)(x, y, z, w) = \psi_1(S)(x, y, Pz, Pw)$$

is curvature-like if and only if $S(x, Py) = S(y, Px)$. Obviously

$$\psi_2(S)(x, y, Pz, Pw) = \psi_1(S)(x, y, z, w).$$

The tensors

$$\pi_1 = \frac{1}{2}\psi_1(g), \qquad \pi_2 = \frac{1}{2}\psi_2(g), \qquad \pi_3 = \psi_1(\widetilde{g}) = \psi_2(\widetilde{g}),$$

where $\tilde{g}(x, y) = g(x, Py)$, are curvature-like, and the tensors $\pi_1 + \pi_2$, π_3 are Riemannian P-tensors.

Let us recall the following statement.

Theorem 1.1. ([2]) *A curvature-like tensor L on 4-dimensional Riemannian almost product manifold is a Riemannian P-tensor if and only if L has the following form:*
$$L = \frac{1}{8}\left\{\tau(L)(\pi_1 + \pi_2) + \tau^*(L)\pi_3\right\}.$$

The curvature tensor R of ∇ is determined by $R(x, y)z = \nabla_x\nabla_y z - \nabla_y\nabla_x z - \nabla_{[x,y]}z$ and the corresponding tensor of type (0,4) is defined as follows $R(x, y, z, w) = g(R(x, y)z, w)$. We denote the Ricci tensor and the scalar curvature of R by ρ and τ, respectively, i.e. $\rho(y, z) = g^{ij}R(e_i, y, z, e_j)$ and $\tau = g^{ij}\rho(e_i, e_j)$. The associated Ricci tensor ρ^* and the associated scalar curvature τ^* of R are determined by $\rho^*(y, z) = g^{ij}R(e_i, y, z, Pe_j)$ and $\tau^* = g^{ij}\rho^*(e_i, e_j)$. In a similar way there are determined the Ricci tensor $\rho(L)$ and the scalar curvature $\tau(L)$ for any curvature-like tensor L as well as the associated quantities $\rho^*(L)$ and $\tau^*(L)$.

It is known the Ricci identity for an almost product structure in the following form

$$R(x, y)Pz - PR(x, y)z = (\nabla_x \nabla_y P)\, z - (\nabla_y \nabla_x P)\, z.$$

Taking into account (1.1) and $\nabla g = 0$, the latter identity implies immediately

(1.7)
$$\begin{aligned} R(x, y, Pz, w) - R(x, y, z, Pw) &= (\nabla_x F)\,(y, z, w) \\ &\quad - (\nabla_y F)\,(x, z, w). \end{aligned}$$

2. Curvature properties of $\overline{\mathcal{W}}_3$-manifolds

Let (M, P, g) is a $\overline{\mathcal{W}}_3$-manifold, i.e. (1.2) is valid.

The first equality of (1.2) can be rewritten in the form

(2.1)
$$(\nabla_y P)\, z = \frac{1}{2n}\left\{[g(y, z) + g(y, Pz)]\,\Omega + (y + Py)\theta(z)\right\},$$

where the vector Ω is defined by $g(\Omega, x) = \theta(x)$. By differentiation of the condition $\theta(Pz) = -\theta(z)$ and taking into account (2.1) and $\theta(y + Py) = 0$, we obtain

(2.2)
$$(\nabla_y \theta)\, Pz = -(\nabla_y \theta)\, z - \frac{\theta(\Omega)}{2n}\,[g(y, z) + g(y, Pz)].$$

By differentiation of the equality $\tilde{g}(y, z) = g(y, Pz)$ we get

(2.3)
$$(\nabla_x \tilde{g})\,(y, z) = F(x, y, z).$$

We differentiate the first equality of (1.2) and take into account (1.7), (2.3) and $\nabla g = 0$. Thus we obtain

$$\begin{aligned} R(x, y, Pz, w) &- R(x, y, z, Pw) = \\ &= [g(y, z) + g(y, Pz)]\,(\nabla_x \theta)\, w - [g(x, z) + g(x, Pz)]\,(\nabla_y \theta)\, w \\ &\quad + [g(y, w) + g(y, Pw)]\,(\nabla_x \theta)\, z - [g(x, w) + g(x, Pw)]\,(\nabla_y \theta)\, z \\ &\quad + [F(x, y, z) - F(y, x, z)]\,\theta(x) + [F(x, y, w) - F(y, x, w)]\,\theta(z). \end{aligned}$$

Thus, bearing in mind the first equality of (1.2), we have

$$
\begin{aligned}
R(x, y, Pz, w) - R(x, y, z, Pw) = \\
= \frac{1}{2n} \big\{ [g(y, z) + g(y, Pz)]\, A(x, w) \\
- [g(x, z) + g(x, Pz)]\, A(y, w) \\
+ [g(y, w) + g(y, Pw)]\, A(x, z) \\
- [g(x, w) + g(x, Pw)]\, A(y, z) \big\},
\end{aligned}
$$

(2.4)

where

(2.5)
$$
A(y, z) = (\nabla_y \theta)\, z - \frac{1}{2n}\theta(y)\theta(z).
$$

Lemma 2.1. *The following statements are equivalent for a $\overline{\mathcal{W}}_3$-manifold (M, P, g):*

   (i) θ *ia a closed 1-form;*

   (ii) $A(y, z) = A(z, y);$

   (iii) $A(y, Pz) = A(z, Py),\ A(Py, Pz) = A(y, z).$

Proof. From (1.2) and (2.2) we obtain

(2.6)
$$
A(y, Pz) = -A(y, z) - \frac{\theta(\Omega)}{2n}\,[g(y, z) + g(y, Pz)].
$$

Equalities (2.5) and (2.6) imply the truthfulness of the lemma.

$\square$

We substitute Pw for w in (2.4). By the obtained equality, (2.5), (2.6) and Lemma 2.1, we get the following

Theorem 2.2. *The following equality is valid for a $\overline{\mathcal{W}}_3$-manifold (M, P, g):*

$$
R(x, y, Pz, Pw) - R(x, y, z, w) =
$$

(2.7)
$$
= -\frac{1}{2n}\left\{ (\psi_1 - \psi_2)(A)(x, y, z, w) + \frac{\theta(\Omega)}{2n}(\pi_1 - \pi_2)(x, y, z, w) \right\}.
$$

According to Lemma 2.1, the 1-form θ is closed if and if $\psi_1(A)$ and $\psi_2(A)$ are curvature-like tensors. In this case, since π_1 and π_2 are also curvature-like tensors, equality (2.7) implies the following

Theorem 2.3. *The 1-form θ is closed on a $\overline{\mathcal{W}}_3$-manifold (M, P, g) if and only if the following equality is valid:*

$$(2.8) \qquad R(x, y, Pz, Pw) + R(y, z, Px, Pw) + R(z, x, Py, Pw) = 0.$$

Corollary 2.4. *If the 1-form θ is closed on a $\overline{\mathcal{W}}_3$-manifold (M, P, g) then the following equalities are valid:*

$$(2.9) \qquad R(Px, Py, Pz, Pw) = R(x, y, z, w), \qquad \rho(Py, Pz) = \rho(y, z).$$

Further we consider a $\overline{\mathcal{W}}_3$-manifold (M, P, g) with a closed 1-form θ. We define a tensor K of type (0,4) by

$$(2.10) \qquad K(x, y, z, w) = \frac{1}{2} \left\{ R(x, y, z, w) + R(x, y, Pz, Pw) \right\}.$$

Preposition 2.5. *If (M, P, g) is a $\overline{\mathcal{W}}_3$-manifold with closed 1-form θ then K is a Riemannian P-tensor and it has the form*

$$(2.11) \qquad K = R - \frac{1}{4n} \left\{ (\psi_1 - \psi_2)(A) + \frac{\theta(\Omega)}{2n} (\pi_1 - \pi_2) \right\}.$$

Proof. Obviously, properties (1.4) and (1.6) are satisfied for K. Moreover, property (1.5) is valid because of (2.8). Therefore, K is a Riemannian P-tensor. Equality (2.11) follows from (2.7) and (2.10). $\qquad \square$

Corollary 2.6. *If (M, P, g) is a $\overline{\mathcal{W}}_3$-manifold with closed 1-form θ then for the Ricci tensor and the scalar curvatures of K the following formulae are valid:*

$$\rho(K)(y, z) = \rho(y, z) - \frac{1}{4n} \left\{ [\mathrm{tr}A + \theta(\Omega)] \left[g(y, z) + g(y, Pz) \right] + 2nA(y, z) \right\},$$

$$\tau(K) = \tau - \mathrm{div}\Omega - \frac{n-1}{2n} \theta(\Omega), \qquad \tau^*(K) = \tau^*,$$

where divΩ *is the divergence of the vector* Ω.

By virtue of Theorem 1.1 and Corollary 2.6 we obtain the following

Theorem 2.7. *If* (M, P, g) *is a 4-dimensional* $\overline{\mathcal{W}}_3$*-manifold with closed 1-form* θ *then the tensor* K *has the form:*

$$(2.12) \qquad K = \frac{1}{8}\left\{\left[\tau + \mathrm{div}\Omega - \frac{\theta(\Omega)}{4}\right](\pi_1 + \pi_2) + \tau^*\pi_3\right\}.$$

Now we will find a necessary and sufficient condition the tensor R to be a Riemannian P-tensor on a $\overline{\mathcal{W}}_3$-manifold with closed 1-form θ.

If R is a Riemannian P-tensor on a $\overline{\mathcal{W}}_3$-manifold with closed 1-form θ then the left-hand side of (2.7) is zero, i.e. we have

$$(\psi_1 - \psi_2)(A) + \frac{\theta(\Omega)}{2n}(\pi_1 - \pi_2) = 0.$$

Then, according to (2.6), it follows

$$(2.13) \qquad [\mathrm{tr}A + \theta(\Omega)]\,[g(y, z) + g(y, Pz)] + 2nA(y, z) = 0.$$

From (2.13) we obtain $\mathrm{tr}A = -\frac{\theta(\Omega)}{2}$ and then again from (2.13) it follows

$$(2.14) \qquad A(y, z) = -\frac{\theta(\Omega)}{4n}\left[g(y, z) + g(y, Pz)\right].$$

Vice versa, let condition (2.14) be satisfied for a $\overline{\mathcal{W}}_3$-manifold with closed 1-form θ. Then

$$\psi_1(A) = -\frac{\theta(\Omega)}{4n}(2\pi_1 + \pi_3), \qquad \psi_2(A) = -\frac{\theta(\Omega)}{4n}(2\pi_2 + \pi_3)$$

and thus (2.7) implies condition (1.6) for R, i.e. R is a Riemannian P-tensor.

Therefore, it is valid the following

Theorem 2.8. *Let* (M, P, g) *be a* $\overline{\mathcal{W}}_3$*-manifold with closed 1-form* θ. *Then* R *is a Riemannian* P*-tensor if and only if condition (2.14) is valid.*

From Proposition 2.5 and Theorem 2.8 we obtain immediately the following

Corollary 2.9. *Let (M, P, g) be a $\overline{\mathcal{W}}_3$-manifold with closed 1-form θ. Then R is a Riemannian P-tensor if and only if $R = K$. In this case, if $\dim M = 4$ the tensor R has the form*

$$(2.15) \qquad R = \frac{1}{8} \left\{ \tau(\pi_1 + \pi_2) + \tau^*\pi_3 \right\}.$$

3. Curvature properties of of $\overline{\mathcal{W}}_6$-manifolds

Let (M, P, g) is a $\overline{\mathcal{W}}_6$-manifold, i.e. (1.3) is valid. Following the approach of the previous section, now we consider problems for a $\overline{\mathcal{W}}_6$-manifold analogous to the considered ones for a $\overline{\mathcal{W}}_3$-manifold.

The first equality of (1.3) can be rewritten in the form

$$(\nabla_y P)\, z = \frac{1}{2n} \left\{ [g(y, z) - g(y, Pz)]\, \Omega + (y - Py)\theta(z) \right\},$$

from where we obtain

$$(\nabla_y \theta)\, Pz = (\nabla_y \theta)\, z - \frac{\theta(\Omega)}{2n} \left[g(y, z) - g(y, Pz) \right].$$

Let A' be the tensor determined by

$$A'(y, z) = (\nabla_y \theta)\, z + \frac{1}{2n}\theta(y)\theta(z),$$

and K is the tensor determined by (2.10).

We establish the truthfulness of the following statements.

Lemma 3.1. *The following statements are equivalent for a $\overline{\mathcal{W}}_6$-manifold (M, P, g):*

(i) θ ia a closed 1-form;

(ii) $A'(y, z) = A'(z, y)$;

(iii) $A'(y, Pz) = A'(z, Py)$, $A'(Py, Pz) = A'(y, z)$.

Theorem 3.2. *The following equality is valid for a $\overline{\mathcal{W}}_6$-manifold (M, P, g):*

$$R(x, y, Pz, Pw) - R(x, y, z, w) =$$

$$= \frac{1}{2n} \left\{ (\psi_1 - \psi_2)(A')(x, y, z, w) - \frac{\theta(\Omega)}{2n}(\pi_1 - \pi_2)(x, y, z, w) \right\}.$$

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Theorem 3.3. *The 1-form θ is closed on a $\overline{\mathbb{W}}_6$-manifold (M, P, g) if and only if the equality (2.8) is valid.*

Corollary 3.4. *If the 1-form θ is closed on a $\overline{\mathbb{W}}_6$-manifold (M, P, g) then the equalities (2.9) are valid.*

Preposition 3.5. *If (M, P, g) is a $\overline{\mathbb{W}}_6$-manifold with closed 1-form θ then K is a Riemannian P-tensor and it has the form*

$$K = R + \frac{1}{4n}\left\{(\psi_1 - \psi_2)(A') - \frac{\theta(\Omega)}{2n}(\pi_1 - \pi_2)\right\}.$$

Corollary 3.6. *If (M, P, g) is a $\overline{\mathbb{W}}_6$-manifold with closed 1-form θ then for the Ricci tensor and the scalar curvatures of K the following formulae are valid:*

$$\rho(K)(y, z) = \rho(y, z)$$
$$+ \frac{1}{4n}\left\{[\text{tr}A' - \theta(\Omega)]\left[g(y, z) - g(y, Pz)\right] + 2nA'(y, z)\right\},$$

$$\tau(K) = \tau + \text{div}\Omega - \frac{n-1}{2n}\theta(\Omega), \qquad \tau^*(K) = \tau^*.$$

Theorem 3.7. *If (M, P, g) is a 4-dimensional $\overline{\mathbb{W}}_6$-manifold with closed 1-form θ then the tensor K has the form (2.12).*

Theorem 3.8. *Let (M, P, g) be a $\overline{\mathbb{W}}_6$-manifold with closed 1-form θ. Then R is a Riemannian P-tensor if and only if the following condition is valid:*

$$A'(y, z) = \frac{\theta(\Omega)}{4n}\left[g(y, z) - g(y, Pz)\right].$$

Corollary 3.9. *Let (M, P, g) be a $\overline{\mathbb{W}}_6$-manifold with closed 1-form θ. Then R is a Riemannian P-tensor if and only if $R = K$. In this case, if $\dim M = 4$ the tensor R has the form (2.15).*

Acknowledgments

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Dobrinka Gribacheva,
Department of Algebra and Geometry
Faculty of Mathematics and Informatics
University of Plovdiv,
236 Bulgaria Blvd
4000 Plovdiv, Bulgaria
e-mail: `dobrinka@uni-plovdiv.bg`

КРИВИННИ СВОЙСТВА ЗА ДВА КЛАСА НА НАВЕЙРА ОТ РИМАНОВИ МНОГООБРАЗИЯ СЪС СТРУКТУРА НА ПРОИЗВЕДЕНИЕ.

Добринка Грибачева

Резюме. Основната цел на настоящата работа е да се получат някои кривинни свойства на многообразия от два класа риманови многообразия със структура на произведение. Това са два основни класа от класификацията на Навейра за риманови многообразия със структура на почти произведение.