A NEW CLASS OF ADAPTIVE FUNCTIONS: PROPERTIES AND APPLICATIONS

Vesselin Kyurkchiev[1,*], Anton Iliev[2, 3], Asen Rahnev[4], Todorka Terzieva[5], Nikolay Kyurkchiev[6, 7]

[1, 2 ,4 ,5, 6] *Faculty of Mathematics and Informatics,*
University of Plovdiv "Paisii Hilendarski",
24, Tzar Asen Str., 4000 Plovdiv, Bulgaria
[3, 7] *Institute of Mathematics and Informatics, Bulgarian Academy of Sciences,*
Acad. G. Bonchev Str., Bl. 8, 1113 Sofia, Bulgaria
Emails: vkurkchiev@uni-plovdiv.bg, aii@uni-plovdiv.bg, assen@uni-plovdiv.bg,
dora@uni-plovdiv.bg, nkyurk@uni-plovdiv.bg,
aiiliev@math.bas.bg, nkyurk@math.bas.bg
* Corresponding author: vkurkchiev@uni-plovdiv.bg

Abstract. In this note we study some properties of a new family of adaptive functions. More precisely, we prove estimates for the "saturation" - d about Hausdorff metric. A large proportion of the observed computer viruses are characterized by rapid growth over a relatively short period of time, after which gradual cumulative saturation usually occurs. This was the reason why we proposed a new family of cumulative functions that has these properties. Numerical examples, illustrating our results using CAS MATHEMATICA are given.

Key Words: a new class of adaptive functions, Hausdorff distance, two–sided bounds, epidemic data "Storm worm".

Introduction

Definition 1. *We consider the following adaptive function:*

$$M(t) = 1 + e^{\beta(e^{-\beta t}-1)}\big(\beta^2 e^{-\beta t}(t + e^{-t} - 1 +$$
$$+ \frac{(2\pi(\cos(0.5\pi t)-e^{-t})+4\sin(0.5\pi t))e^{-t}}{4+\pi^2}) - 1\big) \qquad (1)$$

for $t > 0, \beta > 1$.

Definition 2. [1], [2] *The Hausdorff distance (the H–distance)* [1] $\rho(f, g)$ *between two interval functions* f, g *on* $\Omega \subseteq \mathbb{R}$, *is the distance between their completed graphs* $F(f)$ *and* $F(g)$ *considered as closed subsets of* $\Omega \times \mathbb{R}$. *More precisely,*

$$\rho(f, g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\}, \quad (2)$$

wherein $||.||$ *is any norm in* $\mathbb{R}^2$, *e. g. the maximum norm* $||(t, x)|| = max\{|t|, |x|\}$; *hence the distance between the points* $A = (t_A, x_A)$, $B = (t_B, x_B)$ *in* $\mathbb{R}^2$ *is* $||A - B|| = max(|t_A - t_B|, |x_A - x_B|)$.

In this article we study some properties of the family (1) and prove two-sided estimates for the "saturation" - d about Hausdorff metric.

Some applications are considered.

Main Results

Evidently,

$$\lim_{t \to 0^+} M(t) = 0,$$
$$\lim_{t \to +\infty} M(t) = 1. \quad (3)$$

First, in this Section we give estimates for the "saturation" - d to the horizontal asymptote in the Hausdorff sense by means of function $M(t)$.

We have

$$H(d) := M(d) - 1 + d = 0 \quad (4)$$

The following theorem gives upper and lower bounds for d.

Theorem 1. For $\beta > 2$ the "saturation"–d satisfies the following inequalities

$$d_l := \frac{1}{1.1(1+\beta^2)} < d < \frac{\ln(1.1(1+\beta^2))}{1.1(1+\beta^2)} := d_r. \quad (5)$$

Proof. From $H'(d) > 0$ we conclude that the function H is strictly monotonically increasing.

Consider the function

$$H_1(d) = -1 + (1 + \beta^2)d. \quad (6)$$

From Taylor expansion we obtain $H(d) - H_1(d) = O(d^2)$.

Hence $H_1(d)$ approximates $H(d)$ with $d \to 0$ as $O(d^2)$ (see, Fig.1 – Fig.2).

In addition $H_1,(d) > 0$.

Further, for $b \geq 2$ we have

$$H_1(d_l) < 0,$$
$$H_1(d_r) > 0.$$

This completes the proof of the theorem.

Numerical experiments

Let $\beta = 7$. From the nonlinear equation (4) we find $d = 0.0696615$.

From (5) we have $d_l = 0.0181818$ and $d_r = 0.0728606$ (see, Figure 1).

In the case $\beta = 10$ we find $d = 0.0394604$, $d_l = 0.0090009$, $d_r = 0.0423981$ (see, Figure 2).
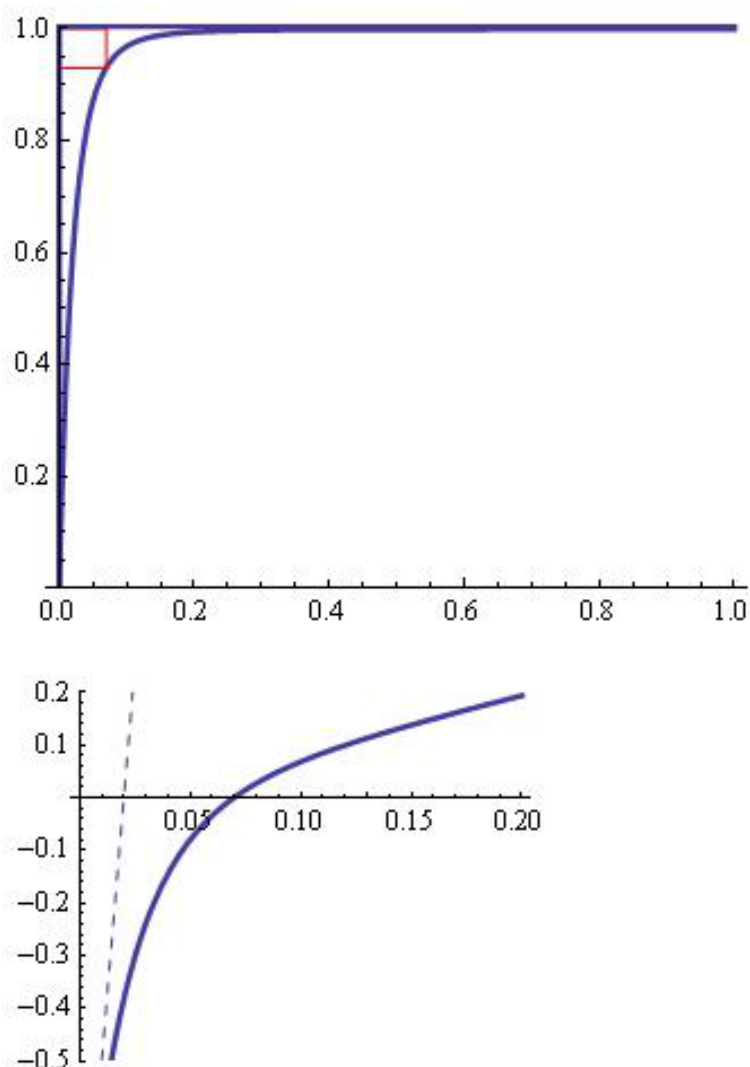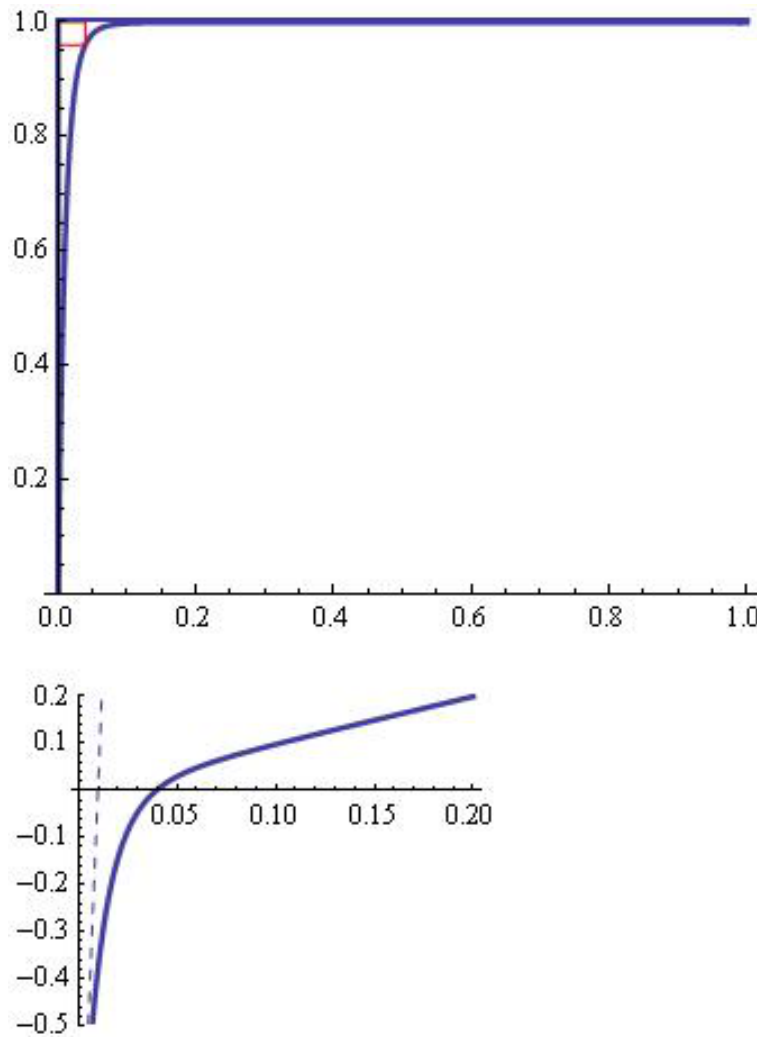

Figure 1. The case $\beta = 7$

Figure 2. The case $\beta = 10$

Some applications

A large proportion of the observed computer viruses are characterized by rapid growth over a relatively short period of time, after which gradual cumulative saturation usually occurs.

This was the reason why we proposed a new family of cumulative functions that has these properties.

Storm worm was one of the most biggest cyber threats of 2008.

1. For the "data_Storm"– normalized (see, [17] for some details) the fitted model $M(t)$ for $\beta = 2.48492$ is depicted on Figure 3.
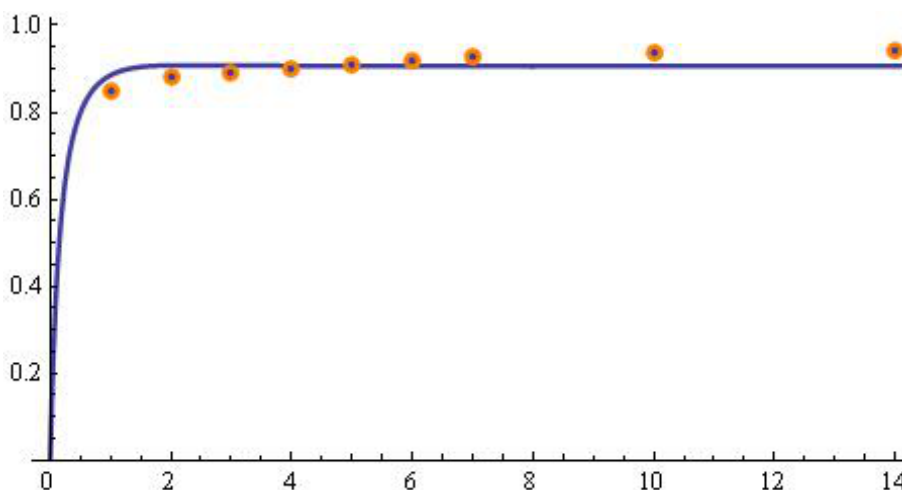
2. For the

$$data\_CDF\_of\_ransoms\_received\_per\_adress\_in\_C_{CL} :=$$

$$\{\{1,0.85\},\{2,0.88\},\{3,0.89\},\{4,0.9\},\{5,0.91\},$$

$$\{6,0.92\}, \{7,0.93\}, \{10,0.935\}, \{14,0.94\}\}$$

the fitted model $M(t)$ for $\beta = 2.374212$ is depicted on Figure 4.
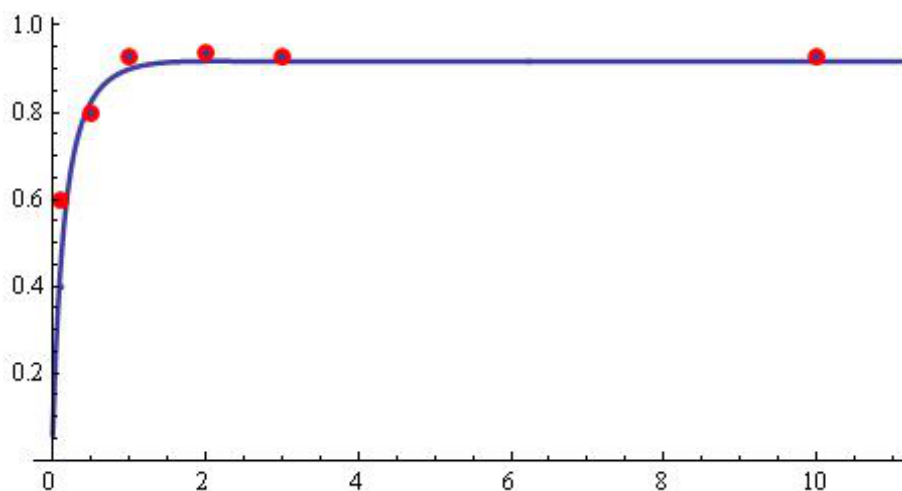
***Figure 3. Epidemic data of Storm worm (normalized)
fitted by $M(t)$ for $\beta = 2.48492$***

***Figure 4. The $data\_CDF\_of\_ransoms\_received\_per\_adress\_in\_C_{CL}$
fitted by $M(t)$ for $\beta = 2.37421$***

Concluding Remarks

These results will be of interest for specialists in this modern scientific branch. Of course, the new adaptive function can be used to analyze specific data in the field of growth theory. We will note that the "sine potential correction" can be used to construct other families of adaptive functions for modeling processes in the field of Computer Viruses Propagation. For some modelling and approximation problems, see [3–26].

Acknowledgment

This work has been accomplished with the financial support by the Grant No BG05M2OP001-1.001-0003, financed by the Science and Education for Smart Growth Operational Program (2014-2020) and co-financed by the European Union through the European structural and Investment funds.

References

[1]   F. Hausdorff, *Set Theory* (2 ed.), New York, Chelsea Publ, 1962.

[2]   B. Sendov, *Hausdorff Approximations*, Boston, Kluwer, 1990.

[3]   P. Szor, *The Art of Computer Virus Research and Defense*, Addison Wesley Professional, 2005, ISBN: 0-321-30454-3.

[4]   C. Zhang, S. Zhou, B. Chain, Hybrid Epidemics-A Case Study on Computer Worm Conficker, *PLoS ONE*, 10, No. 5, 2015, e0127478.

[5]   P. Porras, H. Saidi, V. Yegneswaran, An Analysis of Conficker's Logic and Rendezvous Points, *SRI international technical report*, March 19, 2009.

[6]   Kaspersky Security Bulletin 2015, *Kaspersky Lab*, 2016.

[7]   C. Zou, D. Towsley, W. Gong, On the performance of internet worm scanning strategies, *Performance Evaluation*, 63, No. 7, 2006, 700–723.

[8]   N. Kyurkchiev, A look at the inverse Weibull "adaptive function": properties and applications, *International Journal of Differential Equations and Applications*, 19, No. 1, 2020, 153–165.

[9]   N. Kyurkchiev, *Selected Topics in Mathematical Modeling: Some New Trends (Dedicated to Academician Blagovest Sendov (1932-2020))*, LAP LAMBERT Academic Publishing, 2020, ISBN: 978-620-2-51403-3.

[10] N. Kyurkchiev, Some Intrinsic Properties of Tadmor-Tanner functions. Related Problems and Possible Applications, *Mathematics*, 2020, 8, 1963; doi:10.3390/math8111963.

[11] V. Kyurkchiev, A. Iliev, A. Rahnev, N. Kyurkchiev, *Some New Logistic Differential Models: Properties and Applications*, LAP LAMBERT Academic Publishing, 2019; ISBN: 978-620-0-43442-5.

[12] N. Kyurkchiev, A. Iliev, A. Rahnev, *A Look at the New Logistic Models with "Polynomial Variable Transfer"*, LAP LAMBERT Academic Publishing 2020, ISBN: 978-620-2-56595-0.

[13] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Some software reliability models: Approximation and modeling aspects*, LAP LAMBERT Academic Publishing, 2018, ISBN: 978-613-9-82805-0.

[14] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Nonstandard Models in Debugging Theory (Part 2)*, LAP LAMBERT Academic Publishing, 2018, ISBN: 978-613-9-87794-2.

[15] O. Rahneva, A. Golev, G. Spasov, *Investigations on Some New Models in Debugging and "Growth" Theory" (Part 3)*, LAP LAMBERT Academic Publishing, 2020, ISBN: 978-620-2-66655-8.

[16] N. Kyurkchiev, A. Iliev, A. Golev, A, Rahnev, *Some Non-standard Models in "Debugging and Test Theory" (Part 4)*, Plovdiv, Plovdiv University Press, 2020, ISBN: 978-619-2-02584-7.

[17] A. Iliev, N. Kyurkchiev, A. Rahnev, T. Terzieva, *Some Models in the Theory of Computer Viruses Propagation*, LAP LAMBERT Academic Publishing, 2019, ISBN: 978-620-0-00826-8.

[18] N. Kyurkchiev, A. Iliev, A. Rahnev, T. Terzieva, A new analysis of Code Red and Witty worms behavior, *Communications in Applied Analysis*, 23, No. 2, 2019, 267–285.

[19] A. Iliev, N. Kyurkchiev, A. Rahnev, T. Terzieva, Some New Approaches for Modelling Large-scale Worm Spreading on the Internet. II, *Neural, Parallel, and Scientific Computations*, 27, No. 1, 2019, 23–34.

[20] N. Kyurkchiev, A. Iliev, A. Rahnev, T. Terzieva, A New Analysis of Cryptolocker Ransomware and Welchia Worm Propagation Behavior. Some Applications. III, *Communications in Applied Analysis*, 23, No. 2, 2019.

[21] T. Terzieva, A. Iliev, A. Rahnev, N. Kyurkchiev, On a Powerful Transmuted Odd Log-Logistic-Gumbell Model with Applications to the Theory of Computer Viruses Propagation. V, *Communications in Applied Analysis*, 23, No. 3, 2019, 441–451.

[22] T. Terzieva, A. Iliev, A. Rahnev, N. Kyurkchiev, Comments on a New Hyperbolic Sine-Weibull Model with Applications to the Theory of Computer Viruses Propagation. VI, *International Journal of Differential Equations and Applications*, 18, No. 1, 2019, 137–146.

[23] M. Vasileva, Some Notes on the Omega Distribution and the Pliant Probability Distribution Family, *Algorithms*, 13, No. 12, 2020, 324.

[24] M. Vasileva, O. Rahneva, A. Malinova, V. Arnaudova, The odd Weibull-Topp-Leone-G power series family of distributions, *International Journal of Differential Equations and Applications*, 20, No. 1, 2021, 43–58.

[25] Chesneau, C., Bakouch, H. S., Hussain, T., A new class of probability distributions via cosine and sine functions with applications, *Communications in Statistics-Simulation and Computation*, 48, (8), 2019, 2287–2300.

[26] N. Kyurkchiev, *Some intrinsic properties of adaptive functions to piecewise smooth data*, Plovdiv, Plovdiv University Press, 2021, ISBN: 978-619-202-670-7.