ONLINE EDUCATION: WHAT WORKS IN MATHEMATICS

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Abstract. *Florida State University and University of Helsinki Information technology has the potential to deliver education to everybody by high quality online courses and associated services, and to enhance traditional face-to-face instruction by, e.g., web services offering virtually unlimited practice and step-by-step solutions to practice problems. Regardless of this, tools of information technology have not yet penetrated mathematics education in any meaningful way. This is mostly due to the inertia of academia: instructors are slow to change their working habits. This paper reports on an experiment where all the instructors (seven instructors and six teaching assistants) of a large calculus course were required to base their instruction on online content. The paper will analyze the effectiveness of various solutions used, and finishes with recommendations regarding best practices.*

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2010 Mathematics Subject Classification: 97U50

MATERIALS AND SERVICES OFFERED

LECTURES AND TEN MINUTE TALKS

Online content and services were provided using moodle 1.9. The system was hosted on a Mac OS X Server. Moodle was enhanced by the mathematical quiz system STACK. Materials supporting lectures were offered as pdf files suitable for viewing in class room, and as recorded short presentations (Ten Minute Talks, TMT).

Figure 1 shows a typical traditional way of representing mathematical text in an online environment. The extract is about the definition of the definite integral. Figure 2 illustrates the same topic as represented in the WebALT materials.

The goal in the redesign of the content has been to achieve as compact presentation as possible of the materials. Making the presentation compact, so that the videos based on these presentations can be viewed even on small devices such as smart phones, has resulted in higher overall quality of presentations. Such presentations work very well also on the big screen.

Traditional theatre plays are best in a traditional theater. Movies require different scripts. In the same way printed books presented electronically are not

well suited for small internet devices nor for class-room use in the form of slide shows.

The Definition of the Definite Integral

In this section we will formally define the definite integral and give many of the properties of definite integrals. Let's start off with the definition of a definite integral.

Definite Integral

Given a function $f(x)$ that is continuous on the interval $[a,b]$ we divide the interval into n subintervals of equal width, Δx, and from each interval choose a point, x_i^*. Then the **definite integral of *f(x)* from *a* to *b*** is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$$

The definite integral is defined to be exactly the limit and summation that we looked at in the last section to find the net area between a function and the x-axis. Also note that the notation for the definite integral is very similar to the notation for an indefinite integral. The reason for this will be apparent eventually.

There is also a little bit of terminology that we should get out of the way here. The number "a" that is at the bottom of the integral sign is called the **lower limit** of the integral and the number "b" at the top of the integral sign is called the **upper limit** of the integral. Also, despite the fact that a and b were given as an interval the lower limit does not necessarily need to be smaller than the upper limit. Collectively we'll often call a and b the **interval of integration**.

Let's work a quick example. This example will use many of the properties and facts from the brief review of **summation notation** in the Extras chapter.

Figure 1. A typical online presentation of standard calculus materials. An extract of Paul's Online Math Notes http://tutorial.math.lamar.edu/Classes/CalcI/ /DefnOfDefiniteIntegral.aspx). This is the definition of the Definite Integral.

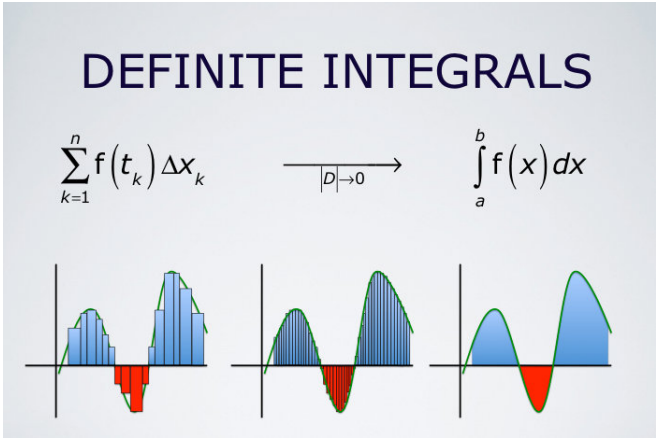

Figure 2. The slide explaining the definition of the Definite Integral as presented in a video included in the WebALT materials.

The academia has a long tradition in printing textbooks and scientific articles. It is hard to think otherwise. A consequence of this fact was that, in this experiment, most instructors chose to either upload their own printed legacy materials to the moodle server, and use that in instruction or use the printed legacy materials of their colleagues. The student use of this materials is given Figure 3.

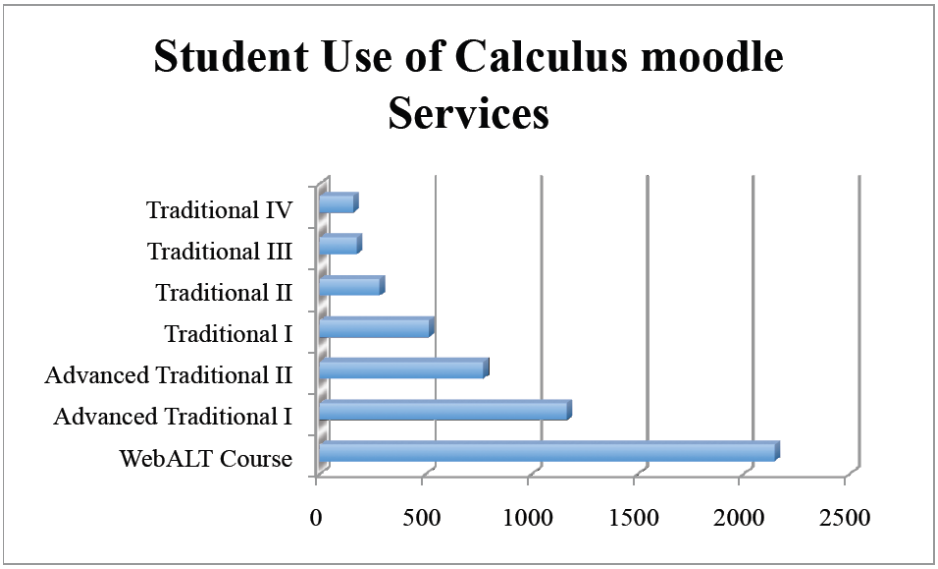

Figure 3. Statistics of student use of moodle materials during week 11 of classes. During this week the topics treated were such that the system did not offer quizzes. The differences in student activities are mostly due to the way instructors were using these resources.

The section "WebALT Course" used the materials (pdf files and videos) conforming to the styled of the slide in Figure 2. The sections entitled "Advanced Traditional I" and "Advanced Traditional II" were using materials developed by the instructors and largely following the presentation described in Figure 1. The sections entitled "Traditional I – IV" were mostly using the materials of the Advanced Traditional Sections. The statistics takes into account the varying numbers of students in sections.

QUIZZES

From the system logs it is obvious that quizzes are the most important part of the service offering (cf. Figure 4). Students embrace them. The three leading sections (WebALT Course, Advanced Traditional I and Advanced Traditional II) used quizzes heavily and allowed students to take quizzes as many times as they wanted (until a certain deadline) with the best result contributing to their grade. The two lowest scoring sections (in Figure 1) used quizzes either not at all or only very little.

Quizzes were offered using the STACK system developed by Chris Sangwin. Problems are based on problem templates. Quiz problems are offered in hundreds of different forms allowing practically unlimited practice. Students like his way of practicing. One reason may be the fact that they are answering to a machine rather

than to their teacher. Hence they do not feel embarrassed of wrong answers. Since most of the problems come with step-by-step solutions, the system provides automatic private instruction.

Week 2		
Deformations of Functions (updated on August 28)	118	Wednesday, 6 October 2010, 05:46 pm (40 days 22 hours)
Introduction to Limits of Functions (updated on August 29)	247	Wednesday, 6 October 2010, 04:29 pm (40 days 23 hours)
Limit Rules (updated on August 31)	285	Thursday, 11 November 2010, 03:22 pm (4 days 23 hours)
One Sided Limits and Limits Rigorously (updated on September 1)	138	Friday, 17 September 2010, 12:40 pm (60 days 3 hours)
Limits 1 (closes on Monday at 4 pm)	688	Sunday, 31 October 2010, 11:50 am (16 days 4 hours)
Limits 2 (closes on Friday at 4 pm)	1109	Saturday, 23 October 2010, 03:52 am (24 days 12 hours)
Recitation Quiz to be taken at home	574	Monday, 8 November 2010, 03:31 pm (7 days 23 hours)
Weekly Joint Discussion Forum -- opens in a new window	36	Thursday, 16 September 2010, 01:36 pm (61 days 2 hours)
Class Week 2 Discussion Forum	851	Wednesday, 22 September 2010, 11:46 am (55 days 4 hours)
Homework due on September 9	139	Saturday, 25 September 2010, 01:38 pm (52 days 2 hours)
Answers to HW 2	41	Monday, 1 November 2010, 09:55 pm (14 days 18 hours)

Figure 4 Week 2 activity report of the WebALT Course. These correspond to the activities of about 90 students. Lectures were viewed typically twice or three times. Quizzes were taken, in average, typically 10 times. Students practiced a lot. The items Limits 1-2 and Recitations quiz were quizzes offered using STACK Services.

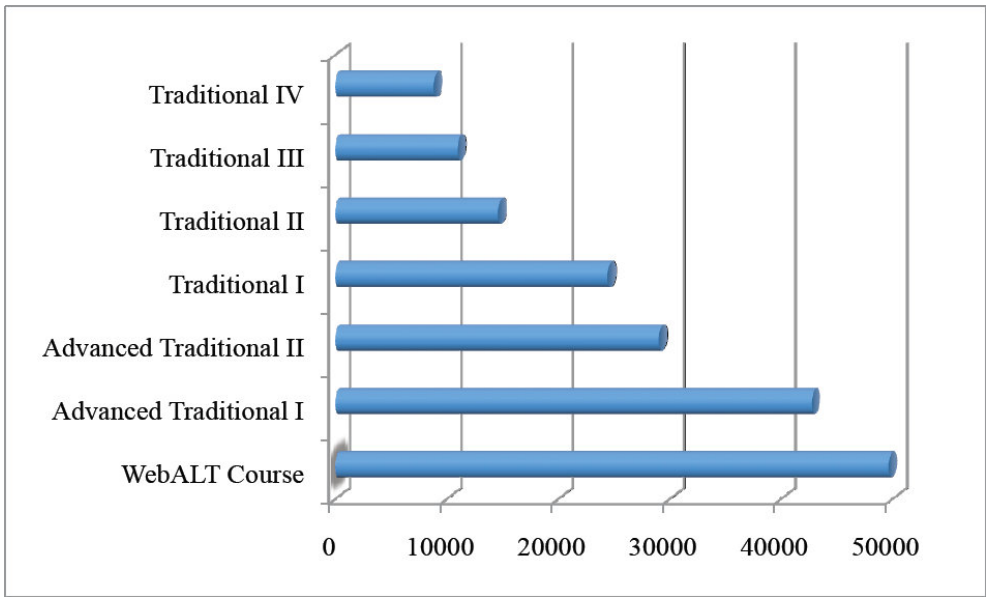

Figure 5 Comparative figures of overall student activity on the calculus moodle server during the first 12 weeks of classes. This statistics takes into account the class size. The three most active classes are the ones using quizzes the most. The section “Traditional IV” has not used quizzes at all.

STUDENT REACTIONS

Student reaction can be analyzed based on the statistics of Figures 1-5. The differences between the ways the system was used by different instructors were big. That was reflected in student use of the system. The bottom line is how the students learn. Comparisons between the learning outcomes cannot be done yet since the courses are still going on during the writing of this report.

CALCULUS INSTRUCTION AT FLORIDA STATE UNIVERSITY

The experiment described above was carried out at Florida State University in fall 2010. At Florida State University calculus is broken into three one semester long courses:

Calculus I, II and III.

Calculus I: Single variable calculus covering limits of functions, continuity, differentiation, applications of differentiation, integration (including the Fundamental Theorem of Calculus), simple integration techniques and first applications of integration (areas, volumes, work). Not covered: concepts like the least upper bound (supremum), and sequences. Enrollment in 2010: 1338 students.

Calculus II: Single variable calculus covering advanced techniques of integration, applications of integration, different coordinate systems, series, Taylor series, and their applications. Enrollment in 2010: 777.

Calculus III: Multivariate calculus. Enrollment in 2010: 600.

This report is about calculus I course at FSU in fall 2010. The enrollment in the beginning of the course was 811 students. By the 11th week of classes 159 students have dropped the class resulting to the enrollment of 652 students. These students were organized in 22 sections (about 38 students initially per section). Each section had its own recitation times. Lectures were offered to three sections of students at the same time in a large lecture hall.

In the past the instruction was based on the use of a textbook. The courses in fall 2010 were based on on-line materials and services only. These were provided to FSU by WebALT Inc., a company operating at the University of Helsinki.

BEST PRACTICES

The author of this paper has used moodle services and content in calculus instruction during the past several years. Previously the instruction has been based on the use of a textbook. In spring 2010 students were told to purchase the standard textbook. Most of them did. In an anonymous survey towards the end of the semester, 86% of the students said that they base their studies mainly or exclusively on the online materials.

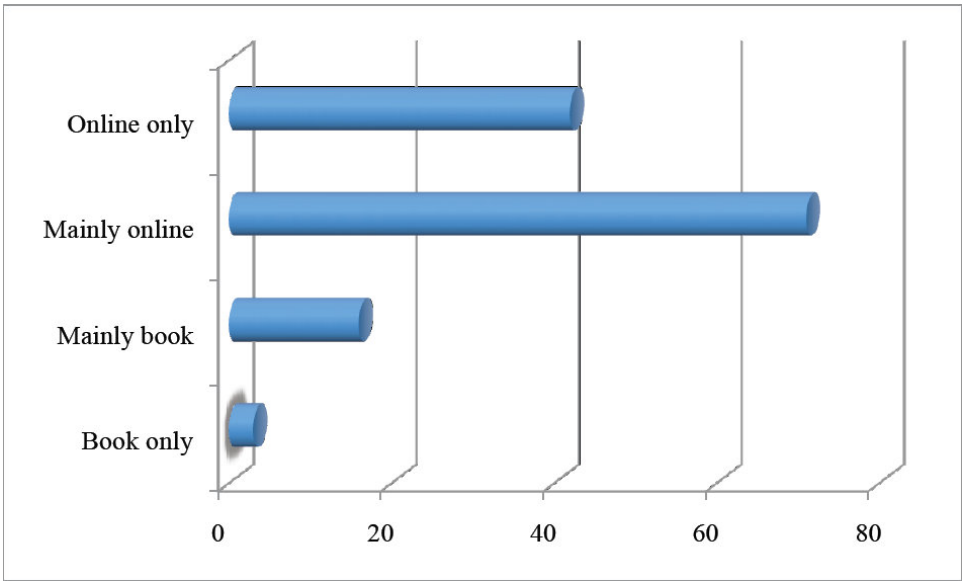

Figure 6 Study habits of 132 calculus II students at FSU in spring 2010. Students purchased the textbook, but did not use it.

The statistics of Figure 6 tell us that the online materials and services are ready to replace the printed textbook. The statistics of Figure 3 and Figure 5 clearly show that students prefer presentations, intended for use with small internet devices, to traditional printed materials presented online. Hence the recommendation is to use and develop content like the slide of Figure 2 rather than content described in Figure 1. The activity reports (Figure 4) further show that students embrace quizzes, which allow virtually unlimited practice with step-by-step solutions as feedback.

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