# ON THE DEVELOPMENT OF STUDENTS' CRITICAL AND REFLECTIVE THINKING IN MATHEMATICS EDUCATION

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**Abstract.** The article examines the essence, pedagogical approaches and challenges related to the development of critical and reflective thinking in students in mathematics education. It analyzes the main manifestations of these activities in the learning process and proposes specific strategies.

***Key Words:*** *critical thinking, reflective thinking, education.*

## Introduction

In today's fast-changing world, critical and reflective thinking are essential for success in both education and life. In mathematics education, these skills help students not only solve problems but also understand concepts, evaluate strategies, and apply knowledge in new contexts. This is especially important in lower secondary school, where students begin to develop more abstract thinking, not just concrete-imagery one. However, traditional teaching often focuses on memorizing formulas, offering limited opportunities to build these skills. As a result, fostering critical and reflective thinking in math education is becoming increasingly relevant and important. The article aims to analyze and present the importance and manifestations of critical and reflective thinking in mathematics education for lower secondary students, and to propose various pedagogical strategies and tools for integrating these skills into the learning process. It also discusses the main challenges faced by both teachers and students in the practical implementation of such approaches.

## Motivation and Related Works

Critical thinking is the ability of a student to analyze, compare, evaluate and draw logical conclusions based on facts, evidence and arguments. It includes skills such as identifying problems, formulating questions, comparing different

viewpoints and making informed decisions. There are many definitions of critical thinking: (1) "Active, persistent and careful consideration of a belief or supposed form of knowledge in the light of the grounds which support it and the further conclusions to which it tends" [1]; (2) "An attitude of being disposed to consider in a thoughtful way the problems and subjects that come within the range of one's experiences" [2]; (3) "A way of thinking – about any subject, content or problem – in which the thinker improves the quality of thinking by skillfully analyzing the structures inherent in thinking and adapting it to clearly articulated intellectual standards" [9]. Halpern [5] and Kurland [7] link critical thinking to developing a purposeful thinking style focused on reasoning, decision-making, and problem-solving, based on logical evaluation of information rather than biases. „Critical thinking enables students to think about their own thoughts and recognize the challenges and opportunities for learning in the most difficult intellectual tasks" [7, p. 113]. According to Lyutskanov, "critical thinking is not meant to tell us what knowledge or validity is, but rather what we have good reasons to be convinced of" [6, p. 15]. Halpern defines critical thinking as „the use of cognitive techniques and strategies that increase the probability of a desirable outcome" [5, p. 22]. This definition sees critical thinking as controllable, justified, and goal-directed in problem-solving and decision-making. We support A. Krasteva's view that such education requires students „to think, to develop and enhance critical thinking skills" [8]. This involves more student-centered activities where critical thinking helps students question knowledge, explore cause-effect relationships, and seek evidence for their conclusions.

Reflective thinking, in turn, is directed toward the awareness and analysis of one's own thought processes. It involves skills such as self-assessment, self-observation, and error correction, as well as recognizing difficulties encountered and reflecting on decisions made. According to Ennis, critical thinking is „reasonable, reflective thinking focused on deciding what to believe or do, based on good information, and resulting in actions or decisions that can be justified as reasonable" [3]. It helps students understand how they arrived at a particular decision, which methods were effective, where they made mistakes, and how they could improve their approach in the future. M. Georgieva points out that reflection and the purposeful application of reflective thinking, can be „a method for developing specific learning skills and fostering student development" [4, p. 12]. It is characterized by „intensive mental engagement, by overcoming difficulties in problem-solving and by self-transformation and the development of the individuals involved." In the course of such activity, the student: orients himself to the task through analysis; activates the necessary knowledge; applies this knowledge in the problem-solving process; formulates hypotheses; applies and varies familiar knowledge and skills; discovers new solutions; tests these solutions; draws conclusions about his own activity. [4, p. 13]. We agree with V. Vasilev that „reflection is the mental tracking and control over the application of the subject's knowledge and qualities in his practical activity" [10, p. 99].

Therefore, we believe that reflection at the thought level helps students modify behavior, increase self-awareness, gain freedom in their environment, and better express and share knowledge.

## Developing critical and reflective thinking

In math education, critical and reflective thinking involves not just applying formulas but analyzing problems, choosing strategies, justifying solutions, and verifying results. For example, solving motion word problems requires understanding conditions, modeling, and assessing solutions. Reflection helps students identify mistakes and improve their approach. We believe that integrating critical and reflective thinking skills enhances academic success and prepares students for life, but requires intentional and well-planned teaching.

The article suggests effective strategies and tools to foster critical and reflective thinking in math education. **(1) Problem-based learning.** The student tackles a real problem without a ready-made formula. For example: *Plan the most efficient route between two towns using different means of transportation with given certain constraints (time, speed).* **(2) Group work and collaborative learning.** Students solve problems, assign roles, and explain strategies. Each group solves a different type of task (e.g., approaching vs. overtaking motion), presents it to the class and answers questions from other groups. **(3) Discussions and justification of solutions.** Students present and defend different approaches of solving a problem (a tabular; an algebraic method). **(4) Open-ended and inquiry-based tasks.** Problems with more than one possible solution or tasks created by students themselves. Example: *Create a problem in which two vehicles meet exactly 1 hour and 30 minutes after departure. Solve it and justify your solution.* **(5) Reflective practices.** Include self-assessment prompts such as: „How did I approach the task?", „How did I choose the method?", „What did I learn from this problem?", „What was the most difficult part, and how did I overcome it?", „What would I do differently next time?". For example, when working on a motion problem with missing or confusing data, a student with critical thinking might ask: „Is there enough information?" or „Can it be solved in more than one way?". Reflective thinking appears in responses such as: „I chose this method because...", or „I realized I had made a mistake because the result didn't make sense in real-world terms". At the end of the lesson, reflective cards (either on paper or in electronic form) can be used with example questions such as: „Which problem-solving method was most useful for me and why?", „What new thing did I learn today?", „What would I do differently next time?". **(6) Formative assessment and self-assessment.** Evaluate the learning process, not just the final result.

This paper presents motion problems for 7th-grade students, through which the proposed ideas for developing critical and reflective thinking can be applied.

This approach fosters logical and analytical skills, deepens concept understanding, and encourages diverse problem-solving strategies. It also boosts creativity and motivation by applying knowledge in practical situations.

**Problem 1.** Two cars start simultaneously from two towns that are 240 km apart, moving towards each other. The first car travels at 70 km/h, and the second at 50 km/h. After how much time will they meet? At what distance from the first car's town will the meeting take place?

**Critical thinking.** The students have to recognize the type of motion – the cars are moving toward each other; analyze the text and extract key information; select an appropriate solving method (a diagram, table, equation or logical reasoning); compare different ideas and check the result for logical consistency. **Reflective thinking.** After solving the problem, students analyze their own activity by asking: "Was I confident that the chosen method was the most effective? Are there other ways to solve the problem? What would I do differently next time?". In this way, they develop the skill to evaluate their own work.

*The problem can be assigned as a group work.*

**Problem 2.** A cyclist starts from town A to town B at a speed of 15 km/h. One hour later, a motorcyclist starts from the same town at a speed of 45 km/h. After how much time will the motorcyclist catch up with the cyclist? How many kilometers will each of them have traveled by the time of the catch-up?

**Critical thinking.** This is a pursuit motion problem with a time delay. **Reflective thinking.** How does the time delay affect the difference in distances? Can we use a diagram? Can you explain the solution to a classmate?

**Problem 3.** Two hikers set off from the same hut along the same trail. The first one starts at 8:00 a.m. at a speed of 4 km/h. The second one sets off later, at 10:00 a.m., at a speed of 6 km/h. At what time will the second hiker catch up with the first one? How many kilometers will they have traveled?

**Critical thinking.** Motion with approach. How is the difference in the starting times accounted for? What is the unknown and how can we express it with an equation? How can we verify the correctness of the solution? **Reflective thinking.** Students understand that when analyzing motion with approach, it is important to consider how far ahead one object is from the other.

**Problem 4.** The distance between towns A and B is 210 km. Two cars start from the towns driving toward each other. One car travels at a speed of 90 km/h, and the other at 60 km/h. One of the cars takes a 15 min break along the way. Is it possible for the cars to meet after 1 hour and 30 minutes? Explain your answer.

**Critical thinking.** An investigative problem with more than one possible solution. Most students will explore only one case. The students must model the situation and not only perform calculations, but also draw logical conclusions.

**Reflective thinking.** The problem is not just about calculation – it requires reasoning about the (im)possibility of the scenario. It's suitable for discussion.

**Problem 5.** Create a motion problem in which one vehicle is chasing another and catches up with it exactly after 2 hours. Define the speeds and the initial time gap. Present the solution and explain your choices.

**Reflection.** This is an open-ended task with multiple possible solutions. The students choose the data. They apply formulas, but also think through and justify their decisions. It's suitable for project or group work.

Incorporating such examples transforms mathematics education from rote learning into the development of complex thinking, reasoning, and self-correction skills. It prepares students for real-life problem-solving, deepens their understanding of concepts, and promotes active learning.

Developing students' critical and reflective thinking in math is a priority but presents challenges for teachers in teaching methods, organization, and student attitudes. We will explore of them and suggest solutions: **(1) Traditional thinking about mathematics as a subject of „exact answers".** Many students perceive mathematics as a subject where they simply need to „find the correct answer". This creates a barrier to developing critical and reflective thinking, which requires time for understanding, questioning, exploring alternatives and analyzing mistakes. The challenge for teachers is to convince students that „thinking on the problem" is no less important than the final result. **(2) Limited time and overloaded curricula.** Mathematics lessons are limited and curricula are dense and focused on acquiring specific knowledge. This creates pressure when incorporating tasks and activities, requiring more time for group work, discussion and reflection. Teachers need to balance curriculum demands with the importance of deep thinking. **(3) Lack of resources and materials.** Suitable teaching resources that include open-ended, exploratory and interdisciplinary tasks aimed at developing thinking skills are not always available. Teachers often have to create or adapt materials themselves, which requires time, creativity and professional initiative. **(4) Diverse student skills and motivation.** In one class students have varying levels of preparation, cognitive maturity and motivation. Some may struggle even with basic skills and feel insecure when faced with non-standard tasks. This challenges teachers to provide accessible, yet stimulating learning for all students. **(5) Fear of mistakes.** Developing critical thinking involves making mistakes, reasoning, reconsidering and searching for better solutions. However, many students fear making mistakes and avoid more complex tasks. Teachers need to create an environment where mistakes are seen as learning opportunities. **(6) Insufficient methodological support.** Most teachers have not undergone targeted training for developing critical and reflective thinking. They mostly rely on self-study, sharing good practices and collegial support. The lack of clear guidelines, examples and systemic support may hinder the application of

such approaches in the classroom. **(7) Assessment focused on the result rather than the process.** Traditional education system assessment is primarily oriented toward the final result. This suppresses students' willingness to experiment with different approaches and think critically. The introduction of formative assessment elements, self-assessment and reflection is advisable.

## Conclusion

Developing critical and reflective thinking requires a trusting environment, encouragement of questioning, positive attitudes toward mistakes, and teachers who ask meaningful questions instead of giving answers. Using suitable problems, reasoning strategies, and self-analysis, students build skills for both classroom and real life. We consider it essential to train future math teachers to foster critical and reflective thinking in their students.

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