MATHEMATICS ACROSS ART, ENGINEERING, AND TECHNOLOGY: A STEAME TRAINING APPROACH FOR STUDENT- AND IN-SERVICE TEACHERS

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Abstract. In this work, we share our experience in conducting training seminars with two groups: student teachers and current teachers, focusing on the STEAME educational approach and project-based learning, as well as the interdisciplinary approach and the role of mathematics in STEAME education across art, engineering, and technology.

Key Words: *STEAME education, project-based learning, mathematics education*

Introduction

STEAME is a multidisciplinary educational approach that integrates science, technology, engineering, arts, mathematics, and entrepreneurship through project-based learning (PBL) [4]. The main goal of this method is to enhance the development of essential skills necessary to meet the challenges of the modern technological world, such as critical thinking, logical reasoning, problem-solving skills, data manipulation, decision-making based on data analysis and interpretation, intellectual curiosity, creativity, innovation, and collaboration skills. In this article, the authors share their experience from their teaching work with students from bachelor’s and master’s programs in the professional field of “Methodology of Teaching in Mathematics, Informatics, and Information Technologies”, as well as in training current teachers [1]. The experience is presented in the context of participation in international educational projects such as FACILITATE-AI (https://facilitate-ai.eu/), BYOD-LEARNING (https://byod-learning.eu/), and currently in STEAME TEACHER FACILITATORS ACADEMY (https://steame-academy.eu/). This article will focus primarily on the interdisciplinary connections of mathematics with art, engineering, and technology in STEAME education.

Math and Multidisciplinary STEAME Approach

Mathematics plays a fundamental role in art, providing structural and aesthetic principles that support the creative process. In modern digital art, mathematical algorithms allow the creation of complex compositions and impressive visual effects. In the field of technology and engineering, mathematics is used as a fundamental language and formalism for modeling, simulations, and optimization. In engineering disciplines, mathematical methods allow for the prediction of system behavior, modelling real-world scenarios, risk assessment, simulate various conditions, process optimization, make informed decisions based on quantitative data, etc. Therefore, the integration of mathematics into the STEAME approach provides the necessary analytical framework for solving complex problems in a multidisciplinary context, and this also determines the urgent need for training future and current teachers to implement STEAME education in schools [5].

In May 2025, two training seminars were held in parallel for students from two bachelor's and two master's programs for mathematics and IT teachers from the Faculty of Mathematics and Informatics, as well as a group of current teachers. A module in both seminars was dedicated to the intriguing connections between mathematics and the arts, and the possibilities for their integration into the STEAME educational environment. Both mathematics and the arts are creative processes that stimulate imagination, encourage originality, and promote innovation and unconventional thinking. The theme of the seminar was "The Beauty of Mathematics (In Search of Harmony and Symmetry)". Students were introduced to various mathematical concepts, forms, and tools used by artists in their creative endeavors. The examples discussed were related to symmetry, proportions, perspective, and geometric shapes that can be found in works of fine art, sculpture, architecture, design, and music.

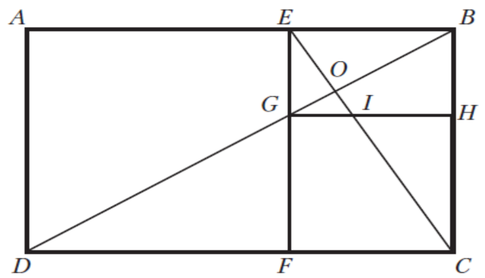
Symmetry in fine art and design was addressed through regular polygons and their connection to regular and semi-regular (Archimedean) tilings of the Euclidean plane, and to Platonic and Archimedean solids. In addition to vertex-to-vertex tessellations, a type of side-to-side tilings, known as Pythagorean tilings, and their connection to the Pythagorean Theorem were also discussed. Popular examples from art and architecture were given. Among them are the tiling on the floor in the painting "Street Musicians at the Door" by the Dutch painter Jacob Ochtervelt (1665) and the tiling on the floor of the St. Vitus Cathedral in Prague (Fig.1). The mosaics of the Dutch designer Maurits C. Escher were also examined. These famous artworks feature various complex shapes and patterns resembling puzzle pieces. Another topic discussed here was the circular geometric symmetry patterns in mandalas. Future teachers were offered ideas for project-based learning, such as covering a floor with various tilings or creating colorful mandalas with a compass and a ruler. This module on symmetry in art and design is suitable for middle school students. Another connection between design and mathematics presented was the use of shapes in 2D and 3D graphic design software (Fig.1). The students were introduced to the basic ideas of curve and surface representation for computer-aided geometric design (CAGD) via implicit, explicit, and parametric equations. Special attention was paid to Bezier curves and B-splines. Free online tools, like Desmos and GeoGebra, were used for demonstrations. Besides equations in Cartesian coordinates, the students were introduced to equations of curves in a polar coordinate system, as this way is more convenient for the representation of shapes having rotational symmetry. In this connection, the superformula of J. Gielis was presented – a mathematical formula that generalizes the circle (ellipse) equation and can be used to generate a wide variety of shapes found in nature [3]. This module is oriented towards high school students in the math profiled training.

Figure 1. Symmetry in the fine art and design

Projective geometry in fine art. Another application of mathematics in fine art was considered through projective geometry and linear projection (central perspective), which Italian Renaissance architects and artists introduced as a method of realistically representing 3D objects on canvas (2D plane) by creating a visual illusion of depth. The students learned about extending the Euclidean plane with points at infinity and a line at infinity. A realistic rendering of floor mosaics containing parallel lines on the plane using points at infinity was considered. Paintings of Renaissance artists like Domenico Veneziano, Raphael, Masolino, and others were examined as examples.

Mathematics and music. The Austrian composer Arnold Schoenberg wrote, “A melody, classic or contemporary, tends towards regularity, simple repetitions and even symmetry”, and another famous composer, Igor Stravinsky, once said, “Musical form is close to mathematics-not perhaps to mathematics itself, but certainly to something like mathematical thinking and relationships”. The students verified the truthfulness of these words by exploring connections between music and mathematics, such as symmetries and proportions. Some of these connections were recognized since ancient times, in the school of Pythagoras. Western music shares structural similarities with mathematics as both use systems built on logic, patterns, and hierarchical relationships, with musical elements like rhythm, harmony, and form often reflecting mathematical principles. The smallest significant structural unit in music is the motif. It is a short musical fragment (idea) recurring in a composition. From motives, themes, and larger structures (phrases, periods, musical forms, etc.), the composition is built. Similar to objects in geometry, musical structures can be subjected to various transformations for the development and variation of musical ideas. Motivic and thematic transformations are essential compositional techniques, not only in imitative polyphony (canons, fugues, etc.). The most commonly used isometric transformations in music are [6]: transposition – horizontal repetition or vertical translation to a different pitch level; inversion – vertical symmetry around a horizontal axes; retrograde symmetry – horizontal symmetry around a vertical axes creating musical palindromes; and retrograde inversion – a combination of retrograde and inversion symmetry (180° rotation). Students were shown examples of these symmetries in classical music compositions. For example, retrograde symmetry and transposition of a motif creating a diatonic sequence can be found in mm. 5-8 of Piano Sonata No. 16, I by W. A. Mozart. The canon "Ma fin est mon commencement" by Guillaume de Machaut, the so-called Crab canon (Canon Cancrizans) by J. S. Bach, and Minuetto al Rovescio from Piano Sonata in A major (Hob. XVI/26 or Landon 41) by J. Haydn represent examples of retrograde symmetry (palindromes) on a compositional scale. Symmetry in music creates a sense of wholeness and order in both spatial and musical structures and forms. This principle is reflected on a larger scale in the repetition of the phrases in a parallel period, the structure of the ternary form (ABA), and the sonata-rondo form (ABACABA).

Harmony in mathematics is defined as the equality or proportionality of the parts to the whole and of the parts to each other. Harmony in music refers to the mathematical relationship between frequencies that create pleasing or consonant sounds when played together. Proportions in music and visual arts were also a topic covered at the seminar. The application of the three principal Pythagorean means – arithmetic, geometric, and harmonic means was considered. The arithmetic and harmonic means are used to generate musical intervals of the overtone (harmonic) series from the two basic intervals, the unison (1:1) and the octave (1:2), in the so-called Ptolemy's pyramid. For example, the pure fifth (3:2) and the pure fourth (4:3) are the arithmetic and harmonic means of the unison and the octave, respectively. The geometric mean, which in music represents a point (tone) equidistant from two other tones, is used to divide the octave into two equal parts, thus creating the ratio of $\sqrt{2}:1$ known as the tritone. In connection with ratios, some tuning systems for musical instruments were discussed – the Pythagorean tuning, Equal temperament, and others. In the same module, the golden section and its generalizations, the so-called metallic means, introduced by V. de Spinadel [2], were considered. The metallic means are defined as the positive root of the equation $x^2 - px - q = 0, p, q > 0$. The second most famous of them, after the golden mean, is the silver ratio ($p = 2,\ q = 1$). Various examples related to the golden ratio from art, architecture and music were presented – from the dimensions of the Parthenon in Athens (Fig.2) and the Notre Dame Cathedral in Paris, the dodecahedron in the painting "The Sacrament of the Last Supper" by Salvador Dali, to proportions in some compositions of J. S. Bach, W. A. Mozart, F. Schubert, C. Debussy, E. Satie and others. The silver ratio can be detected in the so called "roman rectangle" which was used in the late Roman architecture from the 1-2 century. Another famous example is the Horyu-ji Temple in Ikagura in Nara Prefecture, Japan. The question whether the use of the golden or silver ratio by artists and architects is intentional or unconscious was also discussed.



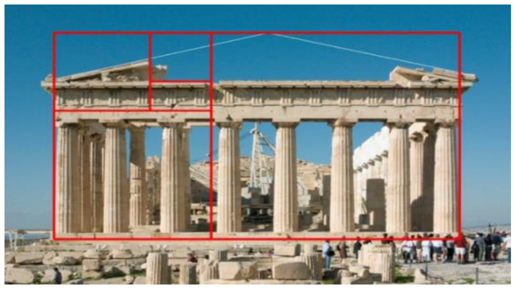


Figure 2. Proportions and harmony in architecture

During the seminars, the students were also introduced to suitable software technologies which can enhance and facilitate the process of teaching and learning mathematics and applying it to natural sciences, economics, entrepreneurship, and the arts in the STEAME classroom. For algebraic computations, a computer algebraic system can be used, such as Wolfram Mathematica, Maple, MATLAB, the statistical language and environment R, etc. For graphical representations of geometric objects given by analytical expressions, dynamic geometric software like GeoGebra, Desmos or The Geometer's Sketchpad can be used. Free Gielis Superformula and Supershape generators can be found online, for example https://www.flowrenderengine.com/shaper-supershapes-generator.html. In the module on music and mathematics, the free music composition and notation software MuseScore and its online sheet music catalogue were used.

Results and conclusions

The parallel experimental training involved 25 students studying to become teachers of mathematics and IT in two bachelor's and two master's programs of the University of Plovdiv, and 10 current STEAME school teachers. After a short motivating presentation, working in groups, they developed and presented STEAME projects on a topic chosen by the group. The joint training of future and current teachers combines theoretical knowledge with practical skills, creating a suitable environment for the exchange of experience, innovation and inspiration. Current teachers contribute with their real practices and challenges from the classroom, and future teachers bring new ideas, modern methods and a fresh perspective. This builds a professional community that encourages collaboration, supports the implementation of innovative technologies, and increases motivation, while improving the quality of teaching and the development of pedagogical skills.

After the end of the training seminar, we asked the participants to fill out a short survey about their level of satisfaction. The results confirm our initial assumptions and expectations: 92.86% of the students and 71.43% of the teachers believe that the considered multidisciplinary aspect is interesting and useful for STEAME training; 85.71% of both groups of trainees were completely personally satisfied with their participation in the seminar and would recommend it to their colleagues.

The authors believe that this good practice is of great importance for both students who will become teachers and in-service teachers. It aligns with the necessary skills and competencies of teachers in modern schools and supports the implementation of the STEAME approach in a real learning environment, in accordance with the national strategy for modern education in Bulgaria.

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