VISUALIZATIONS: THE SYNERGY OF DIGITAL AND MATHEMATICAL COMPETENCIES

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Abstract. From the perspective of mathematical and digital competences, the article explores the role of visualizations, showing the mapping, interaction and influence among them. The main aim is to explore how visualizations can be integrated in a way that simultaneously or complementary develops mathematical and digital competencies. Examples of synergy between two sets of competences are presented and perspective for application in education are discussed. The role of orchestration as a potential perspective area of future research is outlined.

Key Words: *mathematical competence (MC), digital competence (DC), visualization, synergy.*

Introduction

The evolution of mathematical curriculum is analyzed in OECD report [14]. Policymakers challenge how to incorporate data, computational thinking and problem-solving, without overloading students and educators. The report highlights that competencies like numeracy, critical thinking, and problem-solving are widely embedded in mathematics curricula as well as mathematical reasoning and statistics. It is also emphasizing on future oriented competencies such as critical thinking, creativity, and student agency, as well on the potential for personalized learning. In 21st century examples [14] some intersections between DC and MC may be found as computational thinking and coding. The holistic curriculum design should integrate digital tools and cross-curricular themes. From other point of view both approaches, holistic and systematic, are using models, diagrams and simulations that are visualization outcomes. Students with visual learning style should also receive particular focus.

These days, living in algorithmic times, from blockchain to machine learning it is found that these new technologies are redefining the systems of knowledge, causing severe conflicts, and altering the boundaries between what is known and what is knowable [1]. With the emergence of digital technologies and the significance of data in many domains, various visualization techniques are becoming essential to the data processing as well as crucial to the outcomes understanding. Redefining the importance of visualization in science and life necessitates changing how its application in education is perceived as well.

Research in this area highlights the importance and challenges of using visualizations in education. Article [1] explores the possibility of developing visual literacy in children through curiosity and play. In the same vein, study [12] discusses the possibility of students choosing their own artistic and design solutions when visualizing data, what stories they tell through visualizations, and how this encourages their engagement and critical thinking. In the context of Bulgaria, an attempt is made to systematically review and present good practices for developing students' digital competencies in Bulgarian secondary schools, as well as for the professional development of teachers [10]. New technologies, online collaboration, the use of dynamic geometry software, and other STEAM approaches are transforming the learning process through which mathematical competencies are built [4]. These studies outline several key trends and needs, such as the growing need for students to not just be passive viewers of visuals, but to actively participate as creators – through the selection, design, and critique of visual representations; creating a sustainable link between the development of digital competencies and the skills to understand and create visualizations; identifying challenges related to teacher preparation and teaching documents that provide sufficient resources and guidance for integrating visualizations in a way that develops both digital and mathematical competencies.

In this article, we aim to contribute in precisely this direction: to explore how visualizations can be integrated in a way that simultaneously or complementary develops mathematical and digital competencies.

Mapping Mathematical and Digital Competencies

There is an interrelationship between computer science and mathematical knowledge and competences at different levels and aspects. Algorithms, the development of artificial intelligence are based on mathematical models, logical arguments and formal proofs. Table 1 presents the main mathematical competences that are directly applicable in computer science. Within the framework of the PISA (OECD) program, mathematical competence is defined as an individual's ability to formulate, use and interpret mathematics in a variety of contexts - the ability to recognize situations in which mathematics is applicable, to apply concepts, facts, procedures and logical reasoning to solve problems, as well as to evaluate and interpret the results obtained in real-world conditions, in order to make informed decisions as an active and responsible citizen.

There are different systems for representing mathematical competences. A particular emphasis on vision is placed among the three general mathematical competencies – mathematical thinking, mathematical language, and mathematical vision, which includes both abstraction and imagination – in certain interpretations of mathematical competence (China) in the Core Mathematics Competencies (CMC) framework [9].

Digital tools, on the other hand, enhance mathematical competencies in the following ways: problem solving often entails working with large volumes of data and calls for usage of the right algorithms and environments; reasoning and proofs are realized through automated theorem-checking systems; and modeling is transformed into simulation through digital tools. Consequently, the development of computer science specialists depends on the combination of digital and mathematical competencies [11]. This implies the necessity of multidisciplinary training methods that develop both sets of competences concurrently or in tandem.

We selected the mathematical competency sets listed in Table 1 from among the existing sets. Examples of several related visualizations are given in the same table. They are both concept-level and quantitative. Visualizations in Digital Competencies (DC) is presented in Table 2. The next tables 3 and 4 present cross interaction of MC and DC in the context of visualization.

Table 1. Visualizations in Mathematical Competencies (MC)

	MC	**Visualization**
MC1	Symbols and Formalism	numerical values, formulas in graphs and models.
MC2	Communicating	infographics
MC3	Tools and Technologies	software application
MC4	Reasoning	proving algorithm correctness; debugging by logical steps; finding shortest paths; decomposing complex tasks
MC5	Modeling	using graphs to model networks; applying equations to simulate growth; abstracting patterns; generalizing solutions; switching between algebraic and graphical forms.

Table 2. Visualizations in Digital Competencies (DC)

	DC	Visualization
DC1	Information Literacy	searching, filtering and structuring data
DC2	Communication and Collaboration	shared visualizations, posters
DC3	Digital Content Creation	development of interactive graphic objects
DC4	Security	ethical use and sharing of data
DC5	Problem Solving	applying visual methods for analysis

Table 3. MC for DC in the context of visualizations

	DC1	DC2	DC3	DC4	DC5
MC1	data	presenting	diagrams	secure data	equations
MC2	interpreting visuals	collaborative infographics	producing visual reports	ethical communication	visual debate maps
MC3	database visual queries	teamwork visualization tools	coding VR/AR models	safe tool usage, licensing of visual materials	simulation tools
MC4	tracing logical steps in visual data	collaborative debugging visualizations	flowcharts, algorithm animations	transparent reasoning, ethical AI visuals	diagrams, optimization maps
MC5	predictive charts, scenario modeling	team-based modeling, visual outputs	3D simulat., digital twin visualizations	responsible modeling	Dyn. models, visual simulations

Table 4. DC for MC in the context of visualizations

	MC1	MC2	MC3	MC4	MC5
DC1	symbolic charts, dashboards, filtered data	interpreting visuals, reporting graphs	using visualization software for data queries	identifying inconsistencies in datasets	scenario modeling
DC2	shared diagrams, co-created visuals	collaborative infographics, joint reports	teamwork with visualization	group debugging flowcharts	team modeling
DC3	shared diagrams, co-created symb. visuals	collaborative infographics, joint reports	teamwork with visualization tools	group debugging with flowcharts	team modeling with shared vis. outputs
DC4	anonymized visualizations, secure symb.	ethical communication	licensed use of visualiz. tools	reasoning in AI explanations	responsible modeling

DC5	comparative plots for problem choices	debate maps, structured argument visuals	simulation-based problem-solving tools	optimization graphs, network diagrams	visual simulations for complex systems

MC and DC: Synergy and Orchestration

The term “synergy” (συνεργία) means “working together” or “cooperation”. When two or more components work together to provide a higher result than the sum of their separate effects, this is known as the synergistic effect. On the other hand, synergetic research examines how complex systems’ interactions provide order and structure: geometry and design [5], physics and mathematics [6]. Some aspects of synergetic in education are discussed in studies [8, 7, 13].

In Table 5 the synergy dimensions are summarized. Some of them can be decentralized (Blockchain analogy). The triplets Learner-Trainer-System and DC-MC-A2O (actions-to-outcomes) can also be used to describe potential synergy aspects.

Table 5. Synergy Dimensions

	Top-Down	**Horizontal**	**Bottom-Up**	**Transform**
actions	co-map	interact	contribute	redesign
outcome (visual)	rules; privileges; restrictions	participation; influence	brainstorm	restructure; rethink; reflect
agents	control	cooperate	initiate	calibrate

In conclusion, in the article we discussed digital and mathematical competence at the intersection of their visualizations, showing the interaction and synergy between them. Due to the vast processing of data (processes, events, and services), modern technologies are a major influence in the development of trends in thinking. Some software management techniques, such as visualization methods (Kanban), serious games, and lean/agile approaches, have been transferred to other industries. The orchestration, which is used to coordinate automated tasks across several systems into higher-order workflows, also performed this. From one perspective, the orchestration is related to the new Agentic AI method, and it may be used to manage AI agents that have the ability to automate specified and well-described tasks.

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