RUSSIAN-BULGARIAN COOPERATION IN THE FIELD OF WORKING WITH MATHEMATICALLY GIFTED CHILDREN AND TALENTED YOUTH

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Abstract. The paper presents an overview of the results of Russian-Bulgarian projects in the field of mathematical education covering the period from 2005 to 2024. All these projects are related to the project Methods and Information Technologies in Education (MITE), led by S. Grozdev (Bulgaria) and T. Sergeeva (Russia). The cooperation developed in several interconnected directions: organization of international competitions and olympiads among secondary school and university students; sharing experiences in organizing work with gifted children and talented youth; professional development of mathematics teachers; development and implementation of new educational technologies for teaching mathematics; development and joint implementation of programs for supplemental mathematical education for secondary school students. The cooperation between the two countries was very fruitful and contributed to the involvement of mathematically gifted schoolchildren and their teachers in project and research activities, obtainment of new scientific results in the field of mathematics teaching methods, their popularization and implementation.

Key Words: *Russian-Bulgarian cooperation, mathematical education, mathematically gifted children, talented youth*

Chronology and history of Russian-Bulgarian cooperation

Cooperation between Russian and Bulgarian specialists in the field of mathematics education and support of gifted children and talented youth began in 2005. It was a continuation of a long-term collaboration and friendship of two well-known professors in the field of mathematics teaching methods, Professor G. Lukankin (Russia) and Professor I. Ganchev (Bulgaria). The professional and axiological basis of their interaction was taken up by a new generation of Russian and Bulgarian scientists, mathematicians and methodologists. In 2005, an international educational project *Methods and Information Technologies in Education (MITE)* was launched under the leadership of Prof. S. Grozdev from Bulgaria and Prof. T. Sergeeva from Russia.

The objectives of the project were:

- development and testing of new pedagogical technologies;
- identification and support of gifted students;
- professional development of teachers;
- popularization of mathematics;
- expansion of international cooperation in the field of mathematical education.

Numerous educational organizations, scientists and teachers from different regions of Russia, Bulgaria, Kazakhstan and Belarus were involved in the development of the project. One of the key areas of the project is the International Competition “Mathematics and Design”, which will mark its 20th anniversary in 2026. Its purpose is to identify and support gifted students in the field of mathematics and computer science.

The Northern (Arctic) Federal University named after M. V. Lomonosov (NArFU) joined the MITE project in 2010. Its aim was to test the technology for inquiry-based geometry learning using dynamical geometry systems (DGS) developed by Prof. T. Sergeeva and her graduate student I. Serbis of schools in the Arkhangelsk region

In 2014, NArFU launched its own project *Experimental Mathematics* supported by the Dynasty Foundation. The project was focused on introducing inquiry-based learning technology in the style of experimental mathematics into the system of additional mathematical education for schoolchildren. The author of the technology and the project leader was Assoc. Prof. M. Pavlova. Prof. S.

Grozdev was involved in this project as a leading expert in the field of experimental mathematics. One of the elements of this project was an experimental mathematics tournament among students of grades 5–9. It was established in 2015.

In the same year, the International Olympiad in Financial and Actuarial Mathematics was launched. Its founders were the University of Economics – Varna, the Higher School of Insurance and Finance (Sofia), and NARFU (Russia) [4]. The ideological inspirers of this olympiad are S. Grozdev, R. Nikolaev and M. Shabanova.

The MITE project is a constantly evolving organism. New initiatives, initiated by educators and scientists from both countries are emerging in the frames of already existing projects. Thus, in 2022, these efforts resulted in the creation of *Network Project School*. This idea arose from the implementation of international network projects involving secondary school students from Russia and Bulgaria [3]. In 2022, a museum of experimental mathematics was established at the base of NArFU on the initiative of Assoc. Prof. M. Pavlova.

Main results of the projects

One of the most significant results of the MITE project is the involvement of several thousand gifted secondary school and university students from Russia and Bulgaria in research and project activities in the fields of mathematics and computer science over the 20 years since its implementation. Competitions and "*Network Design school*" also contributed to this result.

More and more motivated students are becoming involved in creating exhibits for the Museum of Entertaining Mathematics. These activities help them not only learning mathematical content, but also developing the skills of information retrieval, teamwork, presenting results, etc. [9]

The cooperation between Russian and Bulgarian scientists and educators has made it possible to apply the ideas of inquiry-based learning to mathematics education with DGS. Prof. T. Sergeeva and her graduate student I. Serbis have created a series of workbooks and sets of electronic resources for teaching geometry to students of grades 6-9 through the formulation of research tasks [6], [7]. The effectiveness of her methodology is confirmed by the educational results of school students from several regions of Russia. The technology developed by M. Pavlova is described in the monograph [11]. It is widely used in conducting circle classes with secondary school students and in organization of additional education for gifted children. The project participants established that dynamic geometry systems can also be used when teaching other areas of mathematics. In teaching problem-solving with parameters, economic problems, problems investigating the properties of numerical functions, in teaching problem-solving

for estimating the probability of random events and statistical data analysis. Project participants demonstrated the potential for effective use of these software products in the study of physics, as well as in university-level mathematics courses. Tournament in experimental mathematics is used to informally assess the level of research skills in students [1]. It is held every year for secondary school students from different countries (Belarus, Kazakhstan, Russia). One of the most prolific authors of the tournament tasks is Professor R. Nikolaev. His experience is described in an article [8].

As a result of this joint work, the theory and methodology of teaching and learning mathematics in Russia obtained a new area of research and educational project implementation, "Dynamic Geometry". The theory and methodology of teaching and learning mathematics in Russia obtained a new area of research and educational project implementation, "Dynamic Geometry". Its theoretical foundations are represented by the monographs Prof. S. Grozdev, Prof. T. Sergeeva, Prof. M. Shabanova [5].

The new direction has initiated dissertation research in this area. Under the supervision of Professor M. Shabanova, two PhD theses were successfully defended by T. Shirikova and M. Pavlova.

Experimental mathematics is an interesting field of research not only for scientists but also for secondary school students. Some of the results obtained jointly by Russian and Bulgarian students were published in the journal *Mathematics and Informatics* of the Bulgarian Academy of Sciences [2].

Today, GeoGebra opened new opportunities for learning. GeoGebra involves the use of augmented reality technology and 3D printing. These technologies allow for comparison of geometric models with real objects and visualization of mathematical abstractions, testing the practical feasibility of geometric ideas. The newly opened opportunities are used in the creation and managing of museum exhibits, setting tasks for research projects for students [10].

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