STUDENTS' PERFORMANCE AND REPRESENTATIONAL PICTURES. WHAT DO STUDENTS BELIEVE?

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ABSTRACT

This study investigates students' performance and the role of auxiliary-representational picture in multiplicative situations based on the classification of problems with multiplicative structures proposed by Greer (1992). It also investigates students' beliefs towards the role of pictures. For the purposes of the study, 81 students were asked to complete a questionnaire with eight verbal problems, four of them accompanied by a representational picture. The results indicate that students' performance is higher in tasks of equal group and rectangular area. The results also reveal that the presence of the representational picture do not have a significant impact on students' performance. The data also suggest that students' beliefs towards the role of pictures are not accorded with their performance.

INTRODUCTION

Last decades a great attention has been given on the concept of representation and its role in the learning of mathematics. Visual representations play an important role as an aid for supporting reflection and as a means of communicating mathematical ideas (Agathangelou, Papacosta & Gagatsis, 2008). In certain cases, representations are so closely connected with a mathematical concept, that it is difficult for the concept to be understood and acquired without the use of the particular representation (Gagatsis & Elia). Kaput (1987) suggested that the concept of representation involves the following five components: A representation entity, the entity that it represents, particular aspects of the representational entity, the particular aspects of the entity that it represents that form the representation and finally, the correspondence between the two entities.

A basic discrimination that is pointed out in the region of representations is between internal/mental and external/semiotic representations. Internal/mental representations are mental schemes constructed by individuals in order to represent, explain and understand reality. External/semiotic representations are external symbolic carriers, such as symbols, shapes and diagrams, which aim at representing a specific reality, for example mathematics (Dufour-Janvier, Bednarz & Belanger, 1987). Goldin and Kaput (1996) suggest that there is a dual, two-way relationship between external/semiotic and internal/mental representations.

THEORETICAL BACKGROUND

ICONIC REPRESENTATIONS

Visual representations are a type of external representation that is used extensively in mathematics textbooks, teaching and learning (Agathangelou, Papakosta & Gagatsis, 2008). Many experts mathematicians as Hadamard (1945) and Poincaré (1963), consider imagistic representation as a useful tool in Mathematical Problem Solving (MPS) and frequently attempt to use them (in Stylianou, 2003). Furthermore the NCTM's Principles and Standards for School Mathematics (2000) document include a new process standard that addresses representations and stresses the importance of the use of multiple representations in mathematical learning.

An effort to study the function of pictorial representations was made by Carney and Levin (2002) who proposed five functions that pictures serve in text processing – decorative, representational, organizational, interpretational and transformational. Given Carney and Levin's (2002) five functions that pictures serve in text, Theodoulou, Gagatsis & Theodoulou (2003) proposed a similar categorization for the functions of pictures in Mathematical Problem Solving. Specifically, they suggested that pictures have the following four functions in Mathematical Problem Solving: (a) decorative, (b) auxiliary-representational, (c) auxiliary-organizational and (d) informational.

Decorative pictures do not provide any actual information concerning the solution of the problem. Auxiliary-representational pictures represent part or all of the problem content, but are not necessary to be used in order to solve the problem. Auxiliary-organizational pictures help the students to solve the problem by guiding them to organize the given statements of the problem. Finally, informational pictures provide information that is essential for the solution of the problem; in other words, the problem is based on the picture.

MULTIPLICATION

Several recent studies (Kouba & Fraklin, 1995) have shown that students can solve a variety of multiplicative problems long before formal instruction on the operations of multiplication.

A lot of the research on multiplicative structures deals with the categorization of multiplicative situations .According to Greer (1992) the most important classes of situations involving multiplication include: equal groups, multiplicative comparison, Cartesian product and rectangular area. The equal groups can arise in a variety of ways. Some examples are the mathematization of cases of natural replication, repetitions of a sequence of actions and human practices. Multiplicative comparison verbally expressed by "n times as many as". Cartesian product corresponds to the formal definition of mxn in terms of the number of distinct ordered pairs that can be formed when the first number of each pair belongs to a set with m elements and the second to a set with n elements. The final situation to be considered is rectangular area which refers to the physical arrangement of mxn objects in a rectangular area with m rows and n columns (Greer, 1992).

STUDENTS' BELIEFS

Regarding the role of students' sentiments and attitudes towards the use of pictures in Mathematical Problem Solving, De Bellis & Goldin (2006) supported that the sentimental sector constitutes an internal system of representation. According to their model, the person's ability to solve mathematical problems is based on five types of internal systems of representation which interact. One of these systems is the Sentimental, which refers to the person's sentiments, attitudes, beliefs, values and habits. In the research they conducted, De Bellis & Goldin (2006) found that the sentimental sector can enhance or undermine students' performance in Mathematics.

THE STUDY

PURPOSE

The purpose of this study was to investigate the role of auxiliary-representational pictures in multiplicative problem solving. More specifically the aim of the study was to explore students' performance in multiplicative situations concerning the multiplicative situations and to compare students' performance in multiplicative situations with or without auxiliary-representational pictures. Finally, the study aimed to identify the relation between students' performance in multiplicative situations and the students' beliefs towards the role of representational pictures in MPS.

The research was constructed in order to examine the hypothesis of this study included:

1. Students' performance varies in multiplicative situations of the problem.
2. The auxiliary-representational pictures had no significant effect on students' problem solving performance.
3. Students' beliefs towards the role of auxiliary-representational pictures are according with their performance.

METHODOLOGY

PARTICIPANTS

The sample of the research consisted of 81 fourth grade (9 to 10 years old) students from two elementary schools in two districts of Cyprus.

DATA COLLECTION

In order to collect the data needed for this study, a questionnaire was constructed. The questionnaire consisted of two parts (Part A, Part B). More specifically, the focus was on situations with the placement of the unknown in the third part. Part A consisted of 4 classes of multiplicative situation based on the classification of problems with multiplicative structures proposed by Greer (1992): equal groups, multiplicative comparison, rectangular area and Cartesian product. Part B consisted of 4 classes of multiplicative situation which are accompanied by auxiliary-representational pictures. Also, they were called to answer if they believed that picture helped them to solve the problems.

PROCEDURE

The written questionnaire was administered to the students in usual classroom conditions. Students were given 40 minutes to solve the problems. After the completion of the problem tasks, the questionnaires were collected.

VARIABLES OF THE STUDY

The variables of the study were the following:
Vg: Verbal problem of equal group
Vo: Verbal problem of rectangular area
Vc: Verbal problem of comparison
Vk: Verbal problem of Cartesian product
Rg: Verbal problem of equal group accompanied by auxiliary-representational picture
Ro: Verbal problem of rectangular area accompanied by auxiliary-representational picture
Rc: Verbal problem of comparison accompanied by auxiliary-representational picture
Rk: Verbal problem of Cartesian product accompanied by auxiliary-representational picture
BRg: Students' beliefs about the assistant role of the auxiliary-representational picture in multiplicative problem of equal group
BRo:. Students' beliefs about the assistant role of the auxiliary-representational picture in multiplicative problem of rectangular area
BRc:. Students' beliefs about the assistant role of the auxiliary-representational picture in multiplicative problem of comparison

BRk: Students' beliefs about the assistant role of the auxiliary-representational picture in multiplicative problem of Cartesian product
SVk: Strategy "Problem solving by using verbal solutions in verbal problem of Cartesian product"
SRk: Strategy "Problem solving by using verbal solutions in verbal problem of Cartesian product accompanied by auxiliary-representational picture"

SCORING OF THE TASKS

Problem tasks were scored as follows: 0=wrong answer or no answer, 1=correct answer. Relatively to the question which concerned the use of the pictures by the students affirmative answers were marked as 1 and negative answers were marked as 0. Finally, with regard to the strategy that was used in problems of Cartesian product (SVk, SRk), were scored as follows: 1 when students use the strategy "problem solving by using verbal solutions" and 0 when was absent the particular strategy.

METHOD OF ANALYSIS

For the analysis of the collected data, Gras's Implicative Analysis by using the computer software C.H.I.C. and Microsoft Excel were performed.

RESULTS

Table 1 shows students' performance in four multiplicative situations accompanied or not by auxiliary-representational picture. Students' performance is not altered by the mode of multiplicative situation used except in Cartesian product which has the lowest percentage. It must be noted that similar findings emerged when the problem is accompanied by auxiliary-representational picture. Moreover, the problem of Cartesian product has higher percentage when is accompanied by auxiliary-representational picture whilst the problems of equal group and rectangular area have lower percentage when are accompanied by auxiliary-representational picture.

Table 1. *Students' performance in 4 classes of multiplicative situations accompanied or not by auxiliary-representational picture*

PERCENTAGES OF SUCCESS				
Problems	**Equal Group**	**Rectangular area**	**Comparison**	**Cartesian product**
Verbal	93.83%	93.83%	82.72%	29.63%
Verbal accompanied by auxiliary-representational picture	85.19%	87.65%	83.95%	37.04%

As can been seen in Figure 1, students seem to have negative beliefs against the presence and the role of auxiliary-representational picture in MPS. The lowest percentage refers to the problem of Cartesian product (28,4%). Also, low percentages are observed in problem of rectangular area (37,04%) and comparison (39,51%). In addition, the problem of equal group presents the highest percentage (41,98%). Generally, from Figure 1 it is also evident that students' beliefs are lower regarding their performance in four multiplicative situations.

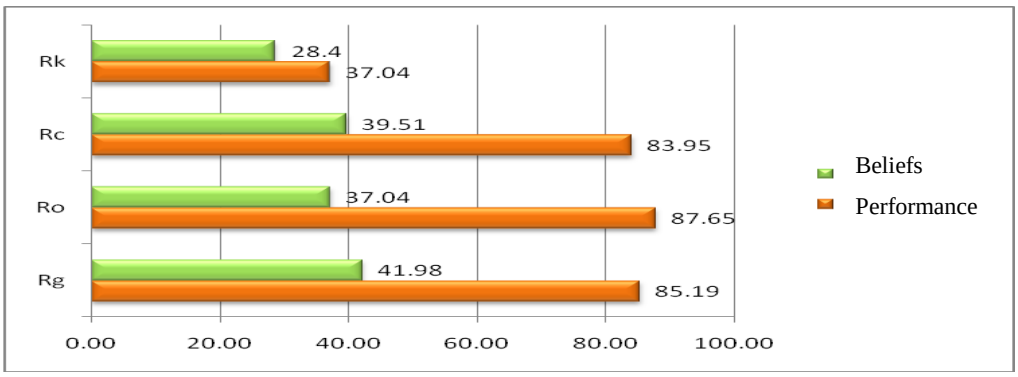

Figure 1. *Students' beliefs and performance*

It must be mentioned that in the problems of Cartesian product (Vk, Rk) most students solve the problem by using the strategy "verbal solution" and not mathematical symbols. Specifically, 46,91% of the students have used this strategy in verbal problems and 50,62% of the students, in verbal problems accompanied by auxiliary-representational picture.

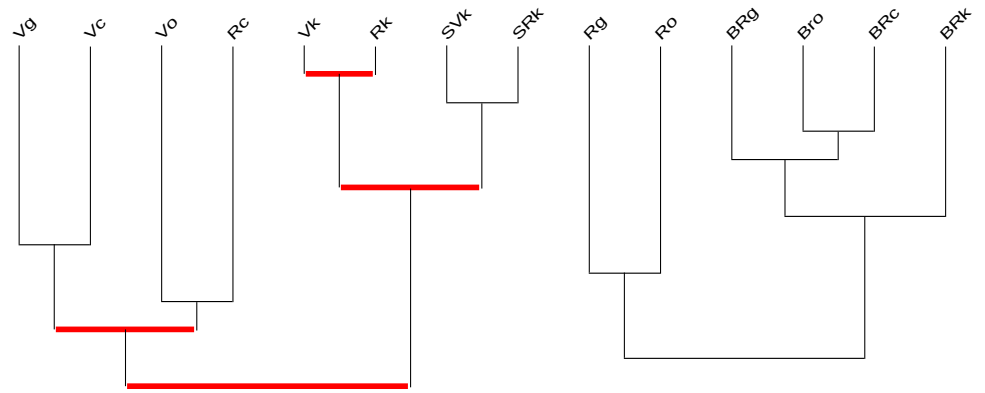

Figure 2: *Similarity diagram*

According to the Similarity Diagram (Figure 2), two groups are clearly distinguished. In the first group, two subgroups are distinguished. The first subgroup consists of the verbal problems of equal group (Vg) and comparison (Vc). Furthermore, the variables Vg and Vc are connected with the variables Vo (verbal problem of rectangular area) and Rc (verbal problem of comparison accompanied by picture). According to the similarity diagram, students justifiably behaved in a similar way when they solved the Verbal problem of equal group and comparison. The same behavior is, also, observed in the verbal problem of rectangular area and verbal problem of comparison accompanied by picture.

The second subgroup consists of the verbal problem of Cartesian product (Vk) and the verbal problem of Cartesian group accompanied by picture (Rk). The fact that tasks of Cartesian product are connected indicates that students deal with these problems with the same way. Moreover, the problems of Cartesian product are connected with the variables SRk and SVk, which represent students' strategy in problems of Cartesian product.

In the second group, also, two subgroups are distinguished. The first subgroup consists of the verbal problems of equal group (Rg) and rectangular area (Ro) accompanied by picture. Consequently, students seem to deal these problems with the same way. Considering the similarity group, the second subgroup consists of the variables which represent students' beliefs towards pictures. (BRg, BRo, BRc, BRk).

As shown in the similarity diagram, students' beliefs towards pictures are connected with their performance in verbal problems of equal group (Rg) and rectangular area (Ro) accompanied by picture.

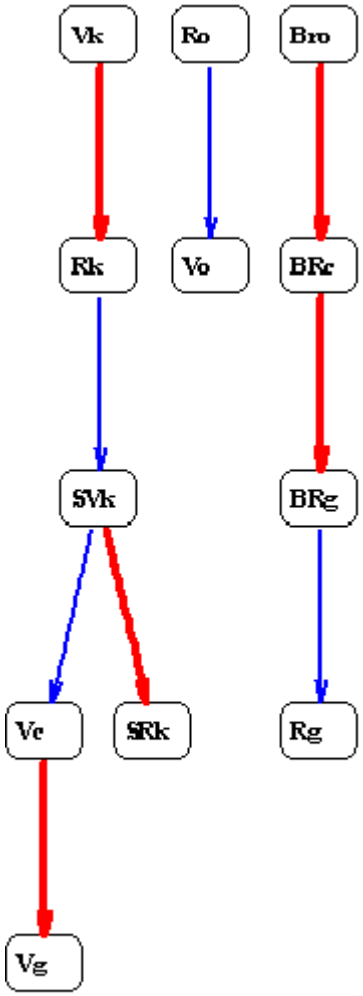

Figure 3: *Implicative Graph*

The Implicative Graph (Figure 3) shows the implications between problem tasks, questions which referred to students' beliefs towards auxiliary-representational picture and students' strategy. According to the implicative graph, three implicative chains are distinguished. In the first implicative chain, it can be concluded that students who solve the verbal problem of Cartesian product (Vk), solve the verbal problem of Cartesian product accompanied by picture (Rk) and use, also, strategy, Problem solving by using verbal solutions, in verbal problem of Cartesian product (SVk) and in verbal problem of Cartesian product accompanied by picture (SRk). It should be also mentioned that the same group of students solve the verbal problem of comparison (Vc) and the verbal problem of equal group (Vg).

The second implicative chain obviously underlines the intra-relational implicative relationship about the tasks of rectangular area. Specifically, students who solve the verbal problem of rectangular area accompanied by picture (Ro), solve, also, the verbal problem of rectangular area (Vo).

As we can see, the third implicative chain shows implications between students' beliefs about the assistant role of the auxiliary-representational picture in problems. Concretely, students who have positive beliefs about the assistant role of the auxiliary-representational picture in task of rectangular area (BRo), have the same beliefs in the tasks of comparison (BRc) and equal group (BRg). Furthermore, the same group of students solves the verbal problem of equal group accompanied by picture (Rg).

DISCUSSION

The gap in the research literature on the role of pictures in Mathematical Problem Solving (Gagatsis & Elia, 2004) contributed to conducting this study. As the results have shown, students' performance in MPS is not affected by the presence of auxiliary-representational pictures, except in the multiplicative situations of equal group and rectangular area. This finding coincides with the findings of a previous study by Gagatsis et al (1999), which showed that different modes of representation, such as pictures, do not always assure successful overlapping of cognitive difficulties in Mathematics. As regards the auxiliary-representational picture, the particular finding is in line with Carney and Levin's (2002) view that auxiliary representations are not necessary for the solution of the problem. It may hinder the process of MPS and it is possible that it requires extra and more complex mental processes. According to Presmeg (1992), pictures may tie thought to irrelevant details, or introduce false information. Consequently, the role of the auxiliary-representational picture is ambiguous. The findings of this research seem to concur with the view that the use of visual representations in mathematics understanding and learning should be conducted with great attention (Seeger, 1998). It is evident that the success percentages in many verbal problems accompanied by a picture are lower than in verbal problems. Thus, reading and using images constitute skills that should not be left to chance, but should be taught and learned systematically (Dreyfus & Eisenberg, 1990, in Agathangelou, Papacosta & Gagatsis, 2008).

The results have also shown that the success percentages are higher in multiplicative situations of equal group and rectangular area than comparison and Cartesian product. One explanation could be that students are more familiar with the multiplicative situations of equal group and rectangular area than comparison and Cartesian product. This finding is in agreement with the results of previous studies (Nesher, 1988: Anghileri, 1989: Kouba, 1989: Mulligan and Mitchelmore, 1997: Christou and Philipppou, 1998, in Gagatsis & Modestou, 2003).

It must be noted that most students solve the verbal problem of Cartesian and the verbal problem of Cartesian product accompanied by auxiliary-representational picture by using verbal solution. A likely explanation is that as a consequence of the absence of symbols, students solve this task by using a verbal solution, and not mathematical symbols.

The results also support that students' beliefs towards the role of pictures are not accorded with their performance in MPS, except in tasks of equal groups and rectangular area. Moreover, students ignored the existence of pictures and their attention was detracted by the numerical data in the problem statement. In the task of Cartesian product, although the students used the picture in order to solve a problem, they claimed that the picture was not useful for solving the problem. (Agathangelou, Papakosta & Gagatsis, 2008).

Further research is needed to investigate the relationship between pictorial representations and the Sentimental system of representation which is proposed by

De Bellis and Goldin (2006), in order to conclude that the sentimental sector can enhance or undermine pupils' performance in Mathematics. Finally, future studies should examine the relation between students' performance and the pictorial representations with a sample of all grades.

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APPENDIX

PART A

1. In a region of African savannah live 4 elephants. An elephant eats 195 kilos of food daily. How many kilos of food consume all elephants in one day?
2. In a shop exist 9 bookshelves. We can put 147 digital disks of music in each bookshelf. How many digital disks exist in the shop?
3. In the confectionery “Sweetie” made 153 chocolates of hazelnut and triple quantity from chocolates of almond. How many are the chocolates of almond?
4. Helen had in her birthday two foods, pizza and hamburger and three drinks, juice, lemonade and water. How many different choices of food-drink can make her guests?

PART B

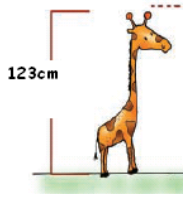
1. The new school year the Primary School of your neighborhood bought 148 boxes with marker pen. Each box had 4 marker pens. How many marker pens did it buy?

Did the picture help you to find the solution of the problem? YES NO

2. Mr John planted 136 lines of roses. Each line had 6 roses. How many roses did Mr John plant?

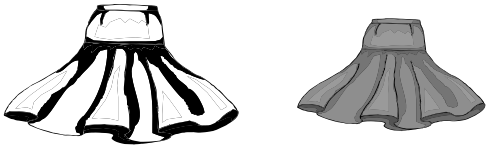
Did the picture help you to find the solution of the problem? YES NO

3. The Zoo of Limassol has two giraffes. The smaller giraffe has height 123 centimeters. The height of biggest is double than smallest. Which is the height of the bigger giraffe?



Did the picture help you to find the solution of the problem? YES NO

4. Maria has two skirts, one black and one white and three tops, one red, one green and one light blue. She wants to go to school combining different colour skirt and top. How many combinations of colour of skirt-top can she make?



Did the picture help you to find the solution of the problem? YES NO