CONGRUENCES IN TRIANGULAR NETS

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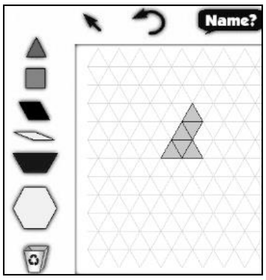
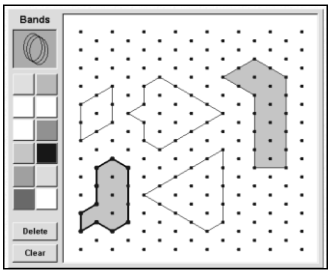
ABSTRACT

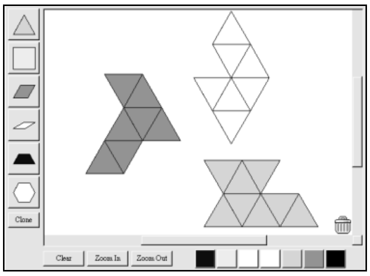
In the history of Geometry development, certain elements of some objects and their properties have been regarded as characteristic for all the objects under consideration. Some of the properties are in such a relation that they divide the objects into types which are defined in a unique way. Abstractions of basic geometric notions from directly used real objects are used in the process of classification. One of them is the resemblance which leads to the notion of congruence in Geometry. The present paper proposes a didactical system of mathematical problems for the propaedeutics of plain congruence.

INTRODUCTION

Since Ancient times, congruence in Nature has been amazing for people. Order and harmony have been referred to symmetry at different levels. The unity of objects, phenomena and knowledge of them is invariably connected with congruence and its break. The regularities give possibilities to formulate hypothesis which should be checked by tools of a corresponding science, while the congruence break leads to the discovery of problem solutions of different nature. Up to the eighth grade of Bulgarian schools, propaedeutics of congruence is carried out by problems which are formulated by everyday life terminology. Congruence touches Literature, Music, Architecture, Nature and s.o.



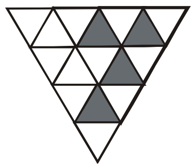


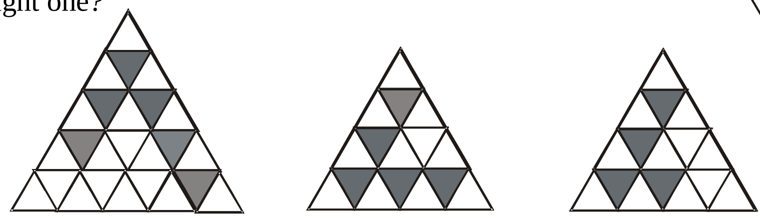


Plain congruence is studied in the eighth grade. Special exercises are needed for the formation of sensitivity to congruence and capability to reason by adapted structures, as well as to give meaning to knowledge and to support it. The use of nets gives possibilities to vary situations of identifying congruent objects and to manipulate them. This paper considers triangular nets of unit equilateral triangles. Examples of problems connected with nets are included in [1]. Computer designed environment with triangular nets are of great importance for students' training. It proposes accumulation of figures, links, structures and modules, as well as capabilities to apply them [2]. The didactical system in the sequel is an example of analogous training which aims at the development of skills to better apprehend congruence and successfully operate with its characteristic elements.

DIDACTIC SET OF PROBLEMS

Problem 1.1. Which of the given three figures is congruent to the right one?

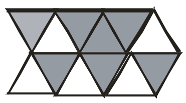




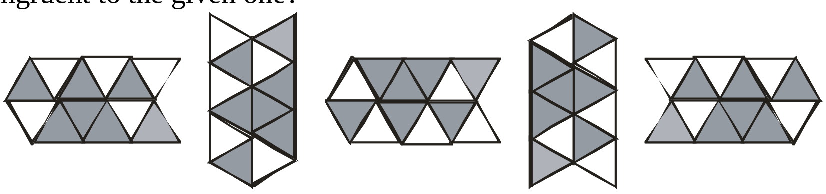
A) B) C)

Answer: C).

Problem 1.2. How many of the five figures

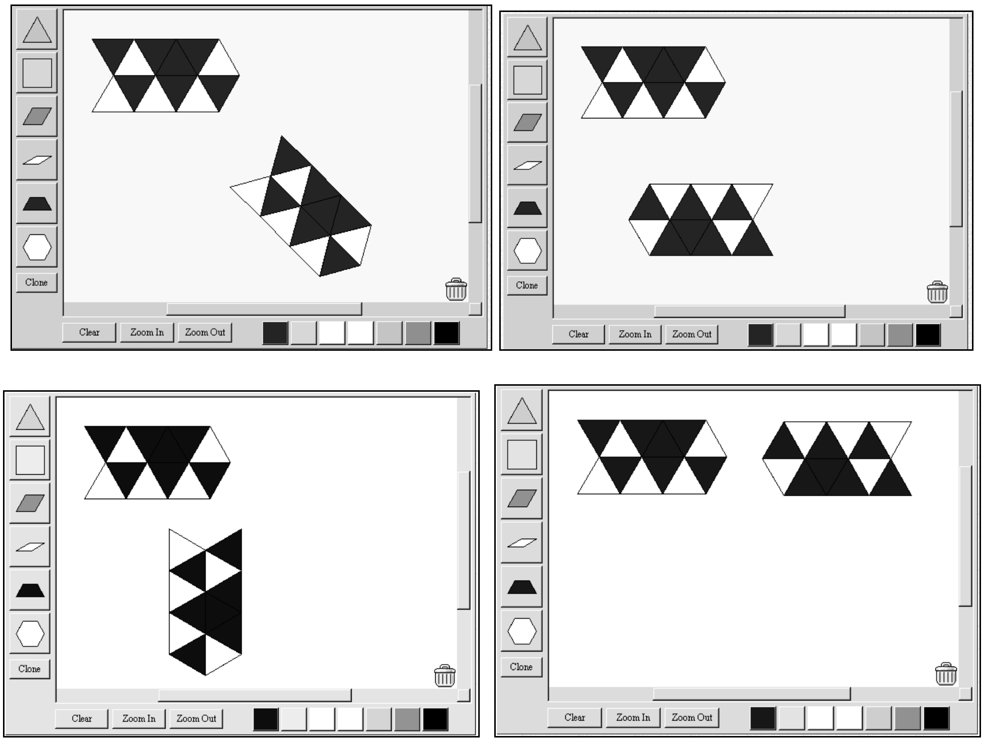


are congruent to the given one?

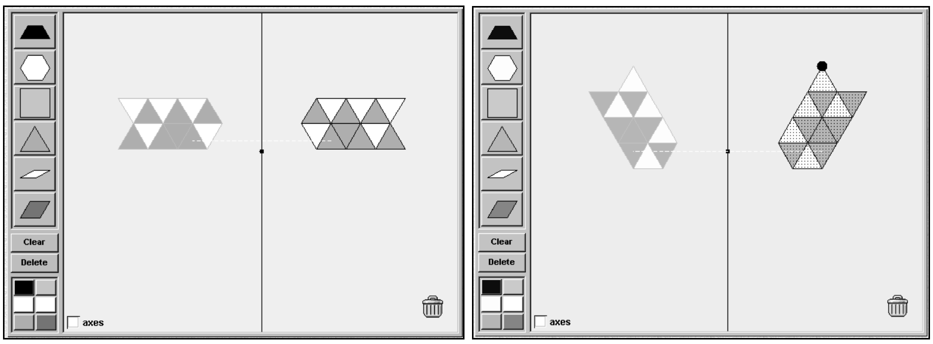


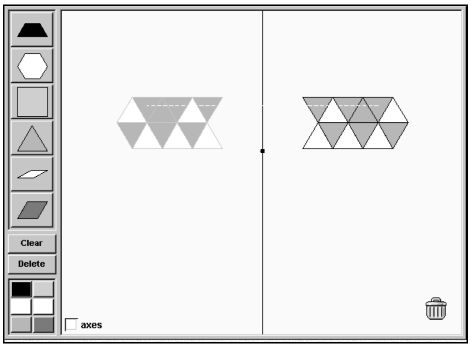
A) 0 B) 1 C) 2 D) 3 E) 4 F) 5

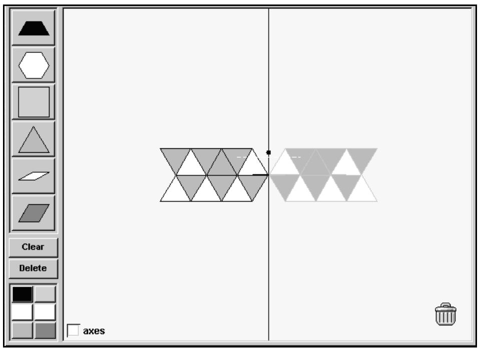
Answer: F). It is enough to use rotation and translation for the first, the second and the fourth figure, while symmetry – for the third and the fifth one. Transformations in the mind should be carried out by students, thus reaching the correct answer. If needed the software *Pattern Blocks* could be applied. It helps in figure constructions, in “cloning”, rotating and displacement of figures.



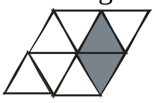
The cited software is not sufficient. Constructions of symmetric figures are necessary, too. Such a possibility could be realized by *Transformations - Reflection*, which constructs a given figure and its symmetric one is displayed together with the axis of symmetry. Displacements and rotations are possible, too

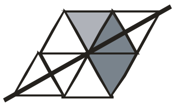


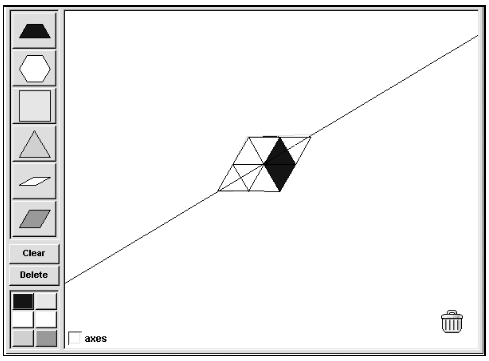


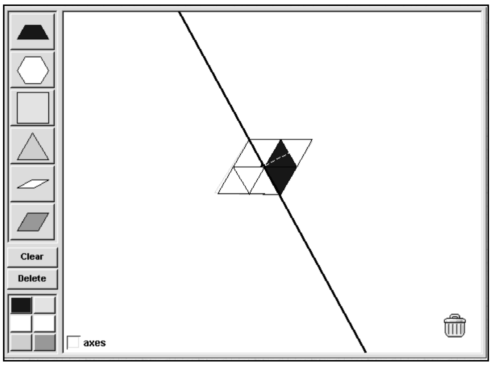


Problem 2.1. What is the smallest number of unit triangles that have to be shaded in order to get an axis of symmetry of the figure below?

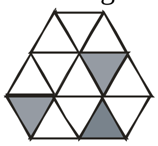


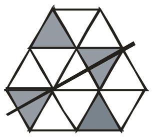
Answer: 1.  The rhombus has two axes of symmetry. For each of the two possible cases the minimal number of triangles to be shaded should be found. In the first case, one unit triangle needs to be shaded, while in the second case it takes two unit triangles. Experimentations are suitable by *Transformations - Reflection* again. What is crucial is the possibility to displace or to rotate the axis of symmetry, which makes the virtual situation closer at a maximal extent to the real one on a sheet of paper. Although overlapping of the original and its image might happen, one could observe unfitting of colors. Enlargement of the software possibilities in problem solving is suitable by the option of color overlapping of figures and use of color combinations. For example, the unit triangles below could be colored in yellow and red but an orange color appears after an overlapping and this leads to the solution.



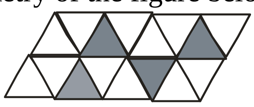


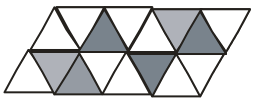
Problem 2.2. What is the smallest number of unit triangles that have to be shaded in order to get an axis of symmetry of the figure below?



Answer: 1.  Apart from the shown one, there are two more axes of symmetry but three unit triangles should be shaded in these cases.

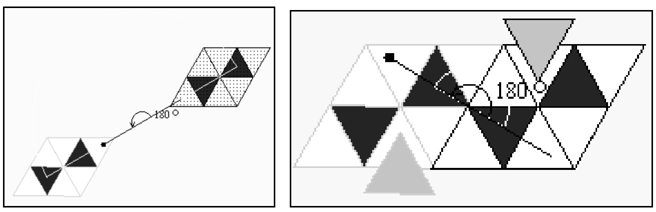
Problem 3.1 What is the smallest number of unit triangles that have to be shaded in order to get an axis of symmetry of the figure below?



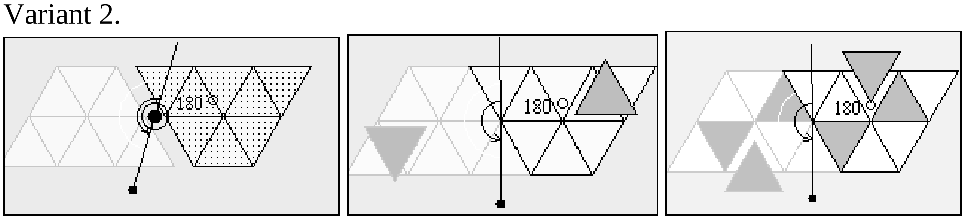
Answer: 2. 

The center of symmetry of the parallelogram is the intersection point of the diagonals. It remains to follow the central symmetric triangles which are not shaded. The central symmetry is not included in the software package under consideration but one could use that it is a rotation at $180°$ in fact. Only a part of the figure could be constructed and checking by a unit triangle is possible. Note that in some cases the fixing of a part is sufficiently difficult. Two variants are proposed which have been used by students.

Variant 1.

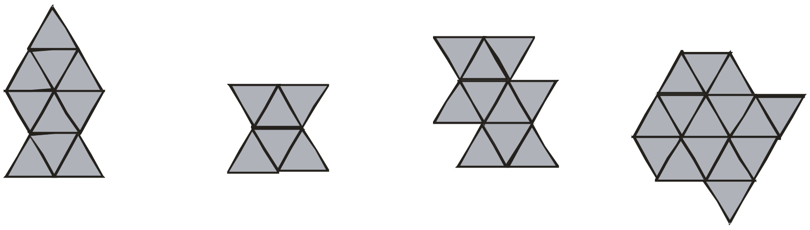


Variant 2.



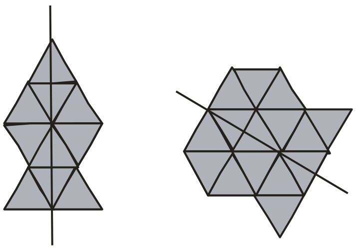
Conjunction of two indications is used in the next problems.

Problem 4.1. How many of the four figures have an axis of symmetry but have no center of symmetry?

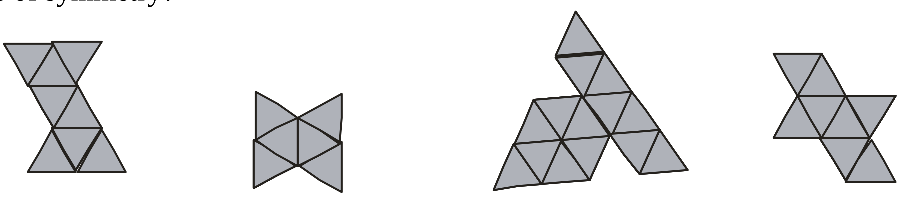


A) 0 B) 1 C) 2 D) 3 E) 4

Answer: C).

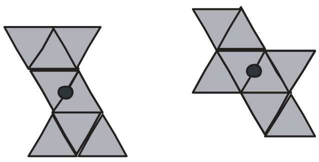


Problem 4.2. How many of the four figures have a center of symmetry but have no axis of symmetry?



A) 0 B) 1 C) 2 D) 3 E) 4

Answer: C).



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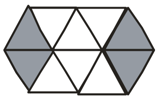
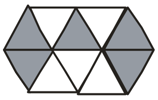
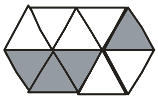
The second figure has a center of symmetry but also an axis of symmetry. The third one has no center of symmetry but it has an axis of symmetry. Observations show that firstly students eliminate the figures with an axis of symmetry and then they check the existence of a center of symmetry.

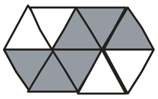
Problem 4.3. What is the smallest number of unit triangles which should be shaded to obtain an axis of symmetry but still the figure should not have any center of symmetry?



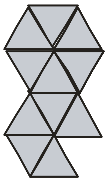
Answer: 1. 

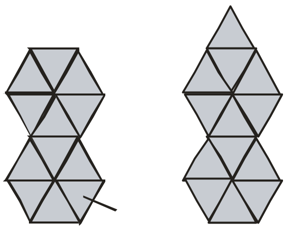
Problem 4.4. What is the smallest number of unit triangles that have to be shaded in order to get a center of symmetry but still the figure should not have any axis of symmetry?



Answer: 2. 

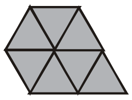
Problem 4.5. What is the smallest number of unit triangles which should be added to obtain an axis of symmetry but still the figure should not have any center of symmetry?



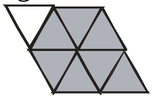
Answer: 2. 

One unit triangle should be added to obtain a horizontal or a vertical axis. Unfortunately, the figure will have a center of symmetry in addition. For this reason, one more unit triangle should be added. Note that the requirement of a minimal number leads to the horizontal and the vertical axis and not to any other one which requires more triangles for a symmetry realization.

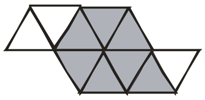
Problem 4.6. What is the smallest number of unit triangles which should be added to obtain a center of symmetry but still the figure should not have any axis of symmetry?



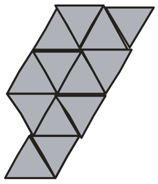
For a center of symmetry, a central symmetric triangle is needed corresponding to the triangle which does not belong to the hexagon. The center of symmetry coincides with the center of the hexagon.

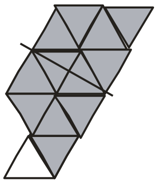


The figure now has an axis of symmetry and for this reason two more central symmetric triangles should be added. This destroys the axis of symmetry. For example:

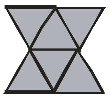


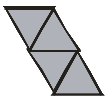
Problem 4.7. What is the smallest number of unit triangles which should be eliminated to obtain an axis of symmetry but still the figure should not have any center of symmetry?



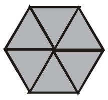
Answer: 1. 

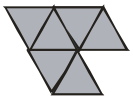
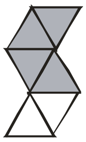
Problem 4.8. What is the smallest number of unit triangles which should be eliminated to obtain a center of symmetry but still the figure should not have any axis of symmetry?



Answer: 2. 

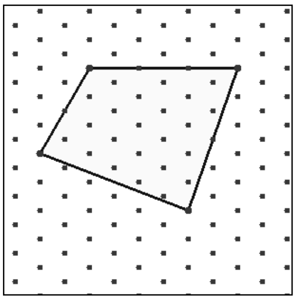
Problem 4.9. What is the smallest number of unit triangles which should be displaced to obtain:
a) an axis of symmetry but still the figure should not have any center of symmetry;
b) a center of symmetry but still the figure should not have any axis of symmetry?



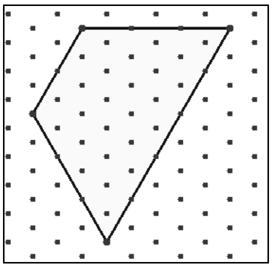
Answer: a) 1;  b) 2. 

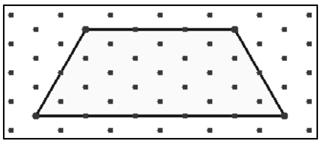
The knots of the triangular net give new possibilities to use studied figures in problem solving connected with congruencies.

Problem 5.1. Change the position of one of the vertices of the quadrilateral to obtain a convex one with an axis of symmetry.



The formulation and the solution of the problem are suitable to be realized by *Geoboard - Isometric*. Here are two possible solutions:

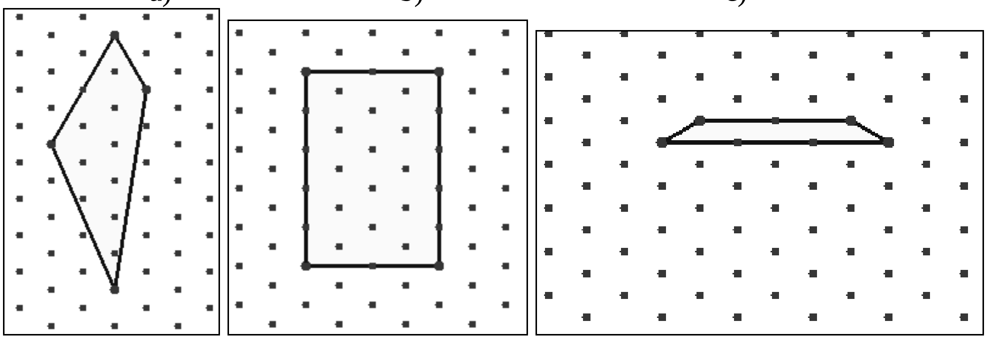




In fact checking should be done for the mediatrices of the sides of the quadrilateral whether a displacement of a quadrilateral vertex to a knot could produce an isosceles trapezoid. Analogous checking of the diagonals could produce a deltoid.

Problem 5.2. Could a displacement of one of the vertices produce a convex quadrilateral with an axis of symmetry?

a) b) c)



The students have received a task to construct figures in a triangular net and to compose problems. Some interesting ideas have been proposed by students from the eighth and ninth grades. Note that rotational symmetry is not included in the obligatory curriculum of Mathematics. At the same time, it is mentioned in other subjects, for example in studying plants. Introduction of different properties and derivation of others are suitable for the non-obligatory curriculum. In all cases, discoveries of congruencies are helpful in problem solving and the preparation of such discoveries is of great importance together with the corresponding environment.

REFFERENCES

[1] Grozdev, S., T. Chehlarova. European Kangaroo. Methodological Approaches. UBM, Sofia, 2008.
[2] Chehlarova, T. Actions by Figures in a Triangular Net, In: Scientific Works of Plovdiv University “P. Hilendarski”, v. 45. 2. Methodics of Teaching, 2008.
[3] *www.nlvm.usu.edu*