TEACHING MATHEMATICS IN HIGHER TECHNOLOGICAL EDUCATION: THE SITUATION IN GREECE

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ABSTRACT

In the present paper the role and status of mathematics is examined for the higher technological education. Our study is illustrated by examples from teaching mathematics at the Graduate Technological Educational Institutes (TEIs) of Greece. Recently reported research in the area of technological education has pointed out that the present situation of the Greek TEIs is similar to what occurs in the majority of technological schools. Thus our conclusions could be applied without significant changes to any department of technological education.

THE ROLE AND STATUS OF MATHEMATICS IN HIGHER TECHNOLOGICAL EDUCATION

All cognitive areas of Technological Education belong to the so called, ***Design Sciences,*** or otherwise ***Sciences of the Artificial***. The scientific status of the Design Sciences was clearly delineated from the corresponding status of the Natural Sciences by the Nobel Prize winner Herb Simon (1970). Indeed, while the Natural Sciences describe and interpret the structure and operation of natural objects, the mission of the Design Sciences is the design and manufacture of artificial objects, having certain desirable properties.

Under this delineation, it becomes evident that Engineering, Graphic Arts and Design, as well as Agricultural and Food Technology, belong to the Design Sciences. It is not hard for someone to understand that Administration and Economics and even the Health Sciences also belong to them. In fact, for the former case, the design of artificial objects concerns the construction of the various economical and management models, while for the latter case it concerns the

treatment and prevention of the various diseases with medicines, operations, vaccines, suitable diet, etc.
According to Simon (1970), the irony is that, while the design should constitute the barometer of all kinds of professional training in the corresponding sciences, the Natural Sciences have considerably displaced the Design Sciences from the curricula of the relevant academic departments, with the climax being in the Polytechnics, the Medical and the Economic Schools, that being included in the general culture and philosophy of the academic rendering, have been changed to a big part to departments of Physics, Biology and of Finite Mathematics, respectively!
On the other hand, mathematics, even if it is not in place to interpret alone the structure of an object, it can however describe it in an explicit and plausible way. Indeed, in many cases a mathematical equation is in place to express in an absolutely evident way something that would need perhaps entire pages of written speech to be expressed. In order to become more explicit, let us give an example from modern Architecture, where the roofs of buildings with wide openings
(e. g. closed stages, swimming pools etc) are usually designed in the form of a saddle, because this type of surface has big resistance to bending (Salvadori and Feller, 1981; paragraph 12.12). Such a surface is called a hyperbolic paraboloid and in a suitably chosen system of coordinates its equation takes the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2cz, \text{ with a, b} > 0.$$

The above equation can be also written as

$$\left(\frac{x}{a} + \frac{y}{b}\right)\left(\frac{x}{a} - \frac{y}{b}\right) = 2cz,$$

wherefrom one easily gets that the corresponding surface is generated by the straight line with equations

$$\left(\frac{x}{a} + \frac{y}{b}\right) = 2\lambda cz \quad \text{and} \quad \left(\frac{x}{a} - \frac{y}{b}\right) = \frac{1}{\lambda},\ 0 \neq \lambda \in R.$$

Thus, mathematics is rightly considered to be the queen, but simultaneously the servant, as well, of the other sciences. In our modern society we must conceive of mathematics as a broad social phenomenon whose diversity of uses and modes of expression is only in part reflected by specialized mathematics, as typically found in university departments of mathematics. This diversity includes mathematics developed and used in science, engineering, economics, computer science, statistics, industry, commerce, art, daily life, and so forth according to the customs and requirements specific to these contents. It becomes therefore evident that nowadays mathematics is necessary to occupy part of the curricula of most departments of Higher Education, either if they concern the Natural Sciences (e.g. Physics, Chemistry, Biology etc), or the Design Sciences, which are under our concern in the present paper. In particular, according to the recent technological developments, this need becomes even greater for many departments of the Design Sciences, as those of engineers, economists etc, where it is necessary for mathematics to constitute a big part indeed of their curricula.

Therefore, the problem that Simon locates, at least for mathematics, is not focused mainly on the quantity of matter that it should be taught at these departments, but on the way in which it will be supposed to be taught, in order that mathematics will be harmoniously tied up with the body of the other courses and become a useful and essential tool for the student (and future scientist) for the study and deeper comprehension of his (her) science.

TEACHING MATHEMATICS AT THE GRADUATE TECHNOLOGICAL EDUCATIONAL INSTITUTES OF GREECE

The Technological Educational Institutes (TEIs) in Greece belong to the Higher Education together with the Universities. The orientation of the TEIs is towards the field of applications and technological research, i.e. the use of the existing scientific knowledge in order to provide new or improved products and services. On the contrary, a more theoretical emphasis is given at the Universities giving access to their graduates towards the basic and (or) the applied research,

The TEIs provide their students a first degree in higher education (at B.Sc. level), as well as postgraduate degrees and diplomas, either independently, or in cooperation with Greek and suitably selected foreign universities.

The TEIs have the following Faculties:

- Administration and Economics
- Technological Applications (wherefrom they graduate technology engineers of various specializations),
- Graphic Arts and Design,
- Health and Caring Professions,
- Agricultural Technology, and
- Food Technology and Nutritiion.

I have been teaching mathematics at the TEIs since 1983, i.e. the year of their foundation with the evolution of the previously existing Centres of Advanced Technological and Professional Education. I have taught at the Faculties of Administration and Economics and of Technological Applications a wide range of topics that starts with Linear Algebra, Vectors and Analytic Geometry, Combinatorial Analysis, Probability, Complex Numbers, Differential and Integral Calculus, Differential Equations etc and is extended up to Numerical Analysis, Mathematics of Finance and Operational Research.

The teaching of mathematics at the TEIs is mainly based on the traditional scheme of theory followed by exercises; thus the weekly schedule for every course is divided into two different parts. The first one comprises the theoretical notions presented by the instructor, while the second part demands the active participation of students in order to apply their knowledge in solving problems.

Students are called to solve and present in class, sometimes working in small groups, exercises relative to their particular interests regarding specific applications of mathematics to technology. A well - equipped computer centre exists in every

Department, so that students have the opportunity to practice using computer packages, and, moreover, become acquainted with modern technological research.

In which way and methods, therefore, does mathematics have to be taught in such an educational environment? In order to answer this question, one has certainly to take seriously into consideration the theoretical frame that we developed in the previous section, about the role that mathematics plays for the Design Sciences, but this is not enough. In fact, in order to give a realistic answer, one has to take also into consideration certain other factors that prevail today in practice in the environment of the TEIs and they considerably influence the teaching of mathematics, but also of other (mainly theoretical) courses.

The first big problem has to do with the fact that the students of the TEIs have actually a non-homogeneous mathematical background. Indeed, graduates from the technical secondary education that have been taught considerably fewer topics of mathematics as compared with the graduates of the positive and technological direction of the general secondary education, also have access (after special examinations) to the TEIs. Still in certain departments of the TEIs, apart from the graduates of the above directions, access is also available to graduates from the theoretical (classical) direction of the general secondary education, a thing that intensifies the problem still more.

This situation is faced casually in certain cases with optional preparatory tuition courses, apart from the regular schedule of teaching, although for arrangements of this type there exist internal disagreements among the teaching stuff of the TEIs (e.g. some believe that in this way the level of studies is degraded).

Therefore, in most cases the instructor of mathematics is compelled to search to find ways to adapt his or her lectures in order to cover the existing voids of some of the students on the one hand, but also to maintain undiminished the interest of the mathematically advanced students on the other hand, a combination that it is really very difficult to be achieved.

With regard to this subject, we should also consider the fact that generally the graduates of the secondary education in Greece nowadays, although they have been taught at high school considerably advanced topics from Mathematical Analysis (up to the integration of a function in one variable), they have serious voids on issues from elementary mathematics, e.g. in the last three years of their studies (Lyceum) they have not been taught the Geometry of Space at all! Imagine therefore, for example, the position of a mathematician, who wants to teach at the department of Renovation and Restoration of Buildings of a TEI (where I am currently teaching in Patras), or may be in a department of Architecture or Civil Engineering of a Polytechnic School of a Greek university, some, even elementary, topics from the theory of surfaces, that constitutes the necessary mathematical background for the design of the roofs of the buildings (Salvadori and Feller, 1981; Chapters 11 and 12), when students have difficulty even to realise that a straight line in space can be described - as a section of two levels - by a linear system of two equations in three unknown variables!

It is also characteristic that even good students in mathematics, although, from what they have learnt in secondary education, know very well the mechanisms of calculation of limits, of derivation and integration of functions etc. and are able to solve difficult exercises concerning the above subjects, they have not absorbed and consolidated properly the corresponding notions of the limit, the continuity, the derivative of a function, of the definite integral etc.

For this phenomenon, one cannot accuse the colleagues of the secondary education, because on the one hand these notions are disproportionately thin and difficult in comparison with the intellectual maturity of the students of this age (17 – 18 years old), and on the other hand, since with the current design of Education in Greece the last two years of the high school constitute substantially the lobby for entrance in higher education (Voskoglou, 2004)), they have to teach in an almost mechanical way in order to prepare their students to face successfully the corresponding General Examinations organized by the Greek Ministry of Education, where a great emphasis is given to exercises.

Also, in many cases, mediocre students fail to apply the theory of secondary trinomial in order to solve exercises of the type: "Study the function $f(x) = x^3 + 2x^2 + x + 7$ with respect to its monotony and find its extreme values", and this is not because they have not learnt it in high school, but probably because it has not been taught to them with the necessary emphasis, as it happened in older days, when we were students.

For all the above reasons, as it results from reports at the annual Conferences on Mathematical Education of the Greek Mathematical Society, from relative discussions etc, most academic teachers would prefer for the graduates of the secondary education to have more solid and completed knowledge of elementary mathematics, than to be so much advanced in Mathematical Analysis (e.g. the integrals could not be taught at high school).

Something else that should also be taken into consideration is that, since the full-time academic staff of the TEIs is not enough to cover all teaching needs and the employment of extra staff is also limited (for economic reasons), the teaching of the theoretical courses is performed to a big audience. Thus, the audience of theory in the mathematical courses of a big T.E.I., as it is for example in Patras, exceeds in many cases 100 students, while in the corresponding tutorials (exercises - applications), where there exists some possibility for the separation of the students in teams, it is usually around 40-50 students.

This has to be connected to the fact that the teaching hours of mathematics in the curricula of the TEIs are relatively limited. For example, in most departments of the Faculty of Administration and Economics, Differential and Integral Calculus is taught for only one semester, that is a half-year period (several topics from Applied Mathematics are also taught over more semesters, like Financial Mathematics, Statistics, Operational Research etc), while at the Faculty of Technological Applications it is usually taught for two or maximum three semesters (in the latter case including Statistics, Numerical Analysis, etc).

It becomes, therefore, evident that under these conditions, it is not so easy for the instructor of mathematics to come in personal contact with students, to discuss with them in comfort, to resolve their queries, and generally to guide them suitably in their studies.

Finally, it must be noted that in certain cases colleagues of other specializations (e.g. economists, engineers etc) avoid, as far as it is possible, the use of mathematics in their courses. This usually happens because, having in mind the weaknesses of their students in mathematics, they try to simplify their teaching material and to present it in a practical form. In this way, however, and despite any efforts of the instructor of mathematics to enrich his or her lectures with applications, mathematics is broken away from the body of the other courses and the students cannot conceive, as they should, the importance that it has for the deeper comprehension of their science. This leads many of them to avoid any serious study of mathematics, as they are convinced that the field of mathematics is totally irrelevant to their technological interests.

In general, the reaction of students towards the learning process in the mathematical courses is too complicated to be described by a single word, but one of its principal features is that a significant percentage of them comfort mathematics with an initial fear, so they attend a course being confident of their potential failure.

How mathematics should be taught at Technological Schools?

After exposing the main factors that influence the teaching of mathematics at the TEIs, let us come back to the question raised in the beginning of the previous section: “In which way and methods does mathematics have to be taught in such an educational environment?”

The easy solution would be to avoid the theoretical proofs by presenting "ready" the corresponding mathematical results to the students and to give emphasis to the examples and applications of these results on the cognitive objects of the various departments.

However, although some times in practice it is almost necessary to apply this method of teaching, we believe that this is not the most advisable way for the general approach to the subject.

Indeed, the graduates of the TEIs - although many of them continue their studies today at postgraduate level and acquire even doctoral degrees - are intended to make carriers mainly as scientists of applications (technologists). This, however, does not mean that they should not acquire the theoretical knowledge that it is essential in order to possess their science, at least in the depth which is necessary in order to be able to check the correctness of their actions in the field of applications.

Accordingly, the proofs of the mathematical theorems should reach and stop up to the point that is indispensable for students to consolidate the corresponding mathematical topic, thus acquiring the ability and the essential dexterities in order to apply it effectively in practice for the study and the deeper comprehension of their science.

For example, the presentation of the equations of the various curves without proofs and the turn of the attention to the solution of exercises and applications only, could not permit the student to conceive the important role that the vectors play for the study of the geometric properties of the various figures with algebraic methods.

On the contrary, if the teacher feels that students have conceived empirically the notion of the limit, i. e. what means for a variable or a function to "tend" to a certain value, it is not necessary to present meticulously all the proofs of the properties of the limits by making use of the analytic definition of the limit. Similarly, if students consolidate rightly the geometric and the natural significance of the derivative, it is not necessary to present all the proofs of the rules of differentiation of functions. Indeed in both cases it is sufficient to present the proof of a simple property and then, by stating "In a similar way one can prove that…", to present all the other properties without proofs.

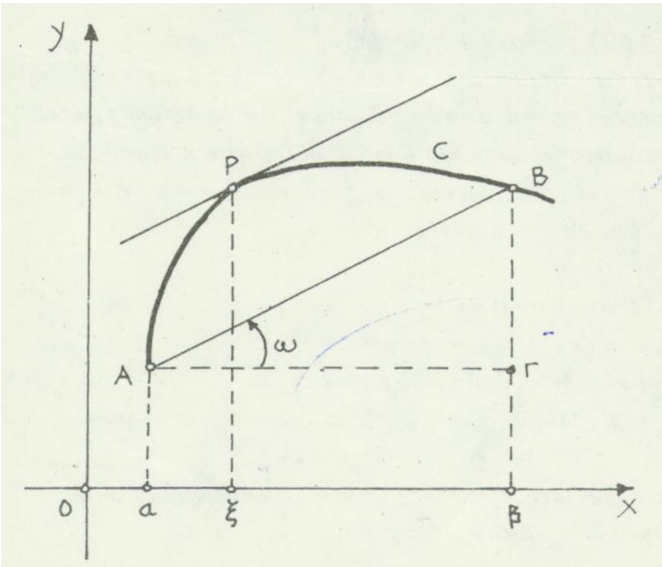

Figure 1

Under the same logic, given function y=f(x), which is the continuous in the interval (a,b), with a and b real numbers, the teacher may present empirically the mean value theorem of the differential calculus (of Lagrange) by making use of the geometric significance of the derivative and "observing" (see Figure 1) that there exists a point, say P(ξ,f(ξ)) on the curve y=f(x), a<ξ<b, such that the tangent of the curve at P is parallel to the chord AB, where A(a,f(a)) and B(b,f(b)), i.e. that

$$\frac{f(b)-f(a)}{b-a} = f'(\xi)\,.$$

In general, through the many years of experience in teaching mathematics at the TEIs, we have reached the conclusion that the excessive mathematical severity and

meticulousness in the proofs disorientate students, many of whom, as we have already seen, do not have the suitable mathematical background for this purpose.
For example, according to the well known method of Newton and Raphson for the approximate calculation of a root of the equation f(x)=0, starting from an approximation, say x_0, of the root, we find a better approximation by the repetitive formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f''(x_n)} \text{ , n=0,1,2,… ,}$$

stopping when the corresponding value of x_n satisfies the precision, say a, that we ask, i. e. when two successive values of x_n differ absolutely less than a.
For the strict proof of the above formula, one has to make use of the Taylor's expression of the function f(x), cutting the terms that contain derivatives of second and greater order. However, a much simpler graphic proof may be presented instead, based again on the geometric significance of the derivative (Tompkins and Wilson**,** 1969; paragraph 3.2).
It is not, however, always easy for the teacher to decide up to which point he or she should proceed with the relative proofs. This depends upon many factors, as they are the cognitive object of the department where he teaches, the time that he or she has at his or her disposal, the level of knowledge base of students, etc. Nevertheless, the previous experience from proportional cases in the past can always direct to the right decision.
The learning process must include ***new technologies*** as a primary tool in order to give representations of mathematical concepts; for example computer packages that are directed to the applications of mathematics, multimedia, use of the Internet, etc. The use of the laboratories, where students have the opportunity to combine their theoretical knowledge with practical work, can help to reduce the distance encountered by students between theory and practice. Working in groups can also encourage students to deal with difficult problems that demand a wide range of knowledge.
Moreover, as for what concerns the teaching process in the classroom, students should be divided into various groups so that, in any group, the level of knowledge base and the interest in mathematics does not display many differences.
Finally, it should be noted that there are topics from applied mathematics with many applications on certain cognitive objects of the TEIs, which are not taught in the required depth, or even worse, they are not included at all in the curricula of the corresponding departments. This should be taken seriously into consideration for the revision of the curricula in future. An example for the former case is the theory of Markov chains, a successful combination of Linear Algebra and Probability theory that enables us to make short and long run forecasts for the evolution of various phenomena (Kemeny & Snell, 1976). An example of the second case is the theory of Fuzzy Sets**,** giving solution to problems where certain definitions do not have explicit limits (e.g. positive development of the stock exchange) and where the classic theory of Probability is not applicable, e.g. decision-making in a fuzzy

environment, statistical evaluation of fuzzy data, etc. (Klir and Folger, 1988,**;** Chapter 15). Even certain basic elements from the Chaotic Dynamics and Fractal Geometry (Mandelbrot, 1983) would be useful to be included in the curricula of certain departments of the TEIs, where they find applications for the confrontation of problems of complexity - non linearity in time and space, respectively, that frequently appear in Physics, Chemistry, Biology, Economics etc.

Recent publications in the area of technological education (Steele, 2003, Walkden and James, 2003)) have pointed out that the present situation at the TEIs is similar to what occurs in the majority of technological schools. Thus, considering also modern learning strategies (Croft and Ward, 2001, Ward, 2003), we believe that the conclusions stated above could be applied without significant changes to any department of technological education.

FINAL CONCLUSIONS

The following conclusions can be drawn from the discussion presented in the paper:

- Mathematics constitutes today a necessary tool for the study and deeper comprehension of the Design Sciences, a big part of which compose the set of all cognitive areas of the Higher Technological Education.
- The main factors that influence the teaching of mathematics at the Greek TEIs are: The non-homogeneous mathematical background of the students of the TEIs, the important voids that the graduates of the secondary education have today in elementary mathematics, the fact that the teaching of mathematics is usually performed to a big audience, while the teaching hours of mathematics in the curricula of the TEIs are relatively limited, and the tendency that quite a number of colleagues of other specializations have to avoid, whenever it is possible, the use of mathematics in their courses.
- The presentation of the theoretical results without proofs and the turn of the attention to the examples and applications is not the most advisable way for the general confrontation of the subject.
- The proofs of theorems must reach and stop up to the point which is indispensable for students to consolidate the corresponding mathematical knowledge, thus acquiring the ability and the essential dexterities in order to use mathematics effectively as a tool for the study and the deeper comprehension of their science.
- The great mathematical severity and meticulousness in the proofs usually disorientate students, many of whom do not have the suitable mathematical background for this purpose. It is not, however, always easy for the instructor of mathematics to decide up to which point to proceed with the relative proofs, because this depends upon many factors. Nevertheless, the experience from proportional cases in the past can always direct to the right decision.

- The use of new technologies (computer packages, multimedia, Internet, etc) can influence students to use mathematics more easily and more effectively. Their utilisation with practical constructions in the laboratories can convince them to face mathematics as a tool and not as an obstacle to their vocational training.
- For the teaching process in the classroom students should be divided to various groups according to the level of their knowledge base and their interest in mathematics.
- There are topics from applied mathematics with many applications on certain cognitive objects of the TEIs that are not taught in the required depth (e.g. Markov Chains), or, even worse, are not included in the curricula of the corresponding departments at all (e.g. Fuzzy Sets, Chaotic Dynamics and Fractal Geometry) and this should be taken seriously into consideration for the revision of the curricula in future.
- Recently reported research has pointed out that the present situation in the Greek TEIs is similar to what occurs in the majority of technological schools. Thus, our conclusions could be applied without significant changes to any department of technological education.

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