NUMBER ANALYSIS AND SOLUTION OF ADDITION AND SUBTRACTION PROBLEMS IN 6-YEAR-OLD CHILDREN

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ABSTRACT

In this paper we present and discuss the findings of an experimental research, which was carried out on 6-year-old children. These particular findings verify the assumption that the infants who were tried in the "approach of number analysis" (research team) have demonstrated a more mature ability of solving narrative problems of the adding type than those who were tried in the usual teaching of the logical-mathematical approach (control group). Moreover, the former group has been able to apply the knowledge of analysis, especially through the analysis-synthesis use of finger patterns in many aspects of the number concept. Therefore, they demonstrated a more mature way of thinking than the children in the control team. All in all, the above results can withstand time as it was proved by the experiment using the same children as subject when they reached the 1st class of elementary school.

INTRODUCTION

Most didactical approaches of numerical notions in pre-school ignore one of the fundamental traits of number: the analysis of number. The older researches lay emphasis on the memorization of the operations (Thorndike, 1922) or the logical and mathematical structures (Piaget & Szeminska, 1941). The recent instructive approaches of early number concepts were formed mainly in the USA during the 1980's and 1990's and despite their differences, they have the oral numeration and the counting strategies as a starting point (Gelman & Gallistel, 1978 – Fuson, 1988 – Hughes, 1986 – Steffe et *all* 1983 – Steffe & Cobb, 1988 – Wright et *all* 2007).

The scientific tendencies mentioned above, overlook the fact that the number is something more than its sole cardinal, ordinal and metric significance. The number is primarily an interesting variety of analysis in smaller numbers and its

reconstruction from them. The analysis and recombining of numbers provide an impetus to imagination, and also motivate children towards permanent investigation, invention of alternative ideas, and creative mathematical expression.

The analysis and synthesis of number are theoretically supported in the work of Piaget and Resnick. The interpretation of numbers with terms of parts and wholes was characterized by Resnick as the biggest mental achievement of early school years (Resnick, 1983). Our perception supports an interpretation for the number which derives partly from the definition of Piaget for the operational significance of number concept (Piaget & Szeminska 1941. Piaget's definition of the number presupposes that the children are able to approach the analyses of given number even if this definition does not presuppose that the children know explicitly these analyses (Brissiaud R., 1994). In addition, Payne and Rathmell (1975) proposed the use of the terms "whole" and "parts" in order to stress emphasis on this significant relation of separation. Various researchers have shown direct or indirect interest in approaches that are related to the decomposition and composition of numbers (Marton & Neuman, 1990 – Sophian & Corgray 1994 – Irwin, 1996 – Hunting, 2003 – Cobb et *all*, 1997).

The main reasons establishing the importance of number analysis on the structure and development of children's numerical knowledge are the following: the multiform and open approach, the invention of reasoning and calculation strategies, the preparation of place value and the solution of problems (Payne & Huinker, 1993 – Baroody, 2004). We consider that an appropriate group of instructive activities could cause and prepare some of the abilities mentioned previously. Indicative activities of this instructive project are the following: activities of partitioning, matching and regrouping by using fingers and other suitable teaching aids that culture provides us with (small collections, parts of the counting board, structured ten, dice, plates of analysis, educational games) and familiar situations of reflection on the teaching aid or other expressive means (verbal recitation of the number word sequence, counting, narrative stories, songs and other multisensory means of the numbers approach), (Kosyvas, 2001).

In our research program in question, the analysis and composition of numbers constitute a fundamental trait for the mathematic development of children. It penetrates into the comprehension of the number concept, as well as of the addition and subtraction. For the comprehension of these operations, the fact that numbers can be separated in smaller ones or be combined in bigger ones is of utmost importance. In each numerical composition, the numbers that are analyzed and composed, codetermine each other, so that a number can result from others through addition or subtraction. This knowledge is connected with the "part-whole" schema and plays an important role in the solution of addition and subtraction problems (Briars & Larkin, 1984 – Riley & Greeno, 1988), since it is primarily inherent in the depth of conceptual relationships of each problem.

We studied narrative problems of addition and subtraction. The pupils that took part in the research solved 10 story problems in total. The question which

occupies us in the present work is: *Will an instructive approach, on the basis of number analysis, bring about a deeper and more mature comprehension into the solution of narrative problems by six-year-old infants in comparison with the usual teaching? Moreover, which strategies are preferred in each case*?

METHOD

The experimental planning includes two groups of children, who attend the second class of kindergarten: the experimental group (or research group) and the control group. The research group attended a new approach of the number concept, which places emphasis on the analysis of number. On the other hand, the control group attended the usual method of the number approach, with activities of logic type - such as classifications, corresponding or serial-ordering - as a key basis.

Ten narrative problems, divided into five types, were given to the children of the total sample (i.e. two problems for each type). Our examination is limited to 6 problems of change and 4 problems of combination (Carpenter & Moser, 1982 – Fisher, 1981, – Resnick, 1989).

The numbers that are included in these narrative problems, both as given numbers and as asked results do not exceed number 8. The table below presents the list of narrative problems given to the children (other numbers that are mentioned in the same scenario are given in brackets):

CATEGORIES	*DICTATION OF THE 10 PROBLEMS*
Change 1: 2 +6=x (3+4=x), increase with the final situation as an unknown quantity	George had 2 (3) marker pens. In his drawing he wants to use more colours. So he is looking for more marker pens and he finds 6 (4). How many marker pens has George got now?
Change 2: 8–6=x (8–4=x), decrease with the final situation as an unknown quantity	Grandfather had 8 (8) hens in the hencoop. However, one day a sly fox got in and ate 6 (4) hens. How many hens remained in the hencoop?
Change 3: 3 +x=7, (2+x=8) increase with the added quantity as the unknown	There were children 3 (2) on the school bus and a few more got on it too. Now there are 7 (8) children on the bus. How many children got on the school bus?
Combine 1: 2+4=x (5+2=x), finding of the whole	Nick has got 2 (5) furry stuffed toys (kittens) and 4 (2) furry stuffed puppies. How many animals Nick has got?
Combine 2: 2+4=x (5+2=x), finding of the part	At Melina's birthday party there were 7 (8) children. 4 (6) of them were girls. How many were the boys?

All children were asked the questions of the ten narrative problems at four different phases:

- Before the application of the two different instructive models for the approach of number concept (pre-test or examination A),
- after the completion of the first phase of courses (intermediary test or examination B),
- after the expiry of all courses at the end of school year (first post-test or C), and
- at the beginning of the new school year in the first class of primary school (second post-test or D).

The experimental research was carried out during the school year 1992-93 in 22 kindergarten classes with a total of 258 pupils. More specifically: During the measurement (in October), our instructive method was used for teaching a hundred and twenty nine (129) children (75 boys, 54 girls) from 11 classes of the Corinthian prefecture, aged from 5*6 (5 years and 6 months) to 6*6 (average=6,02, while 51,9% of the children were older than 6).

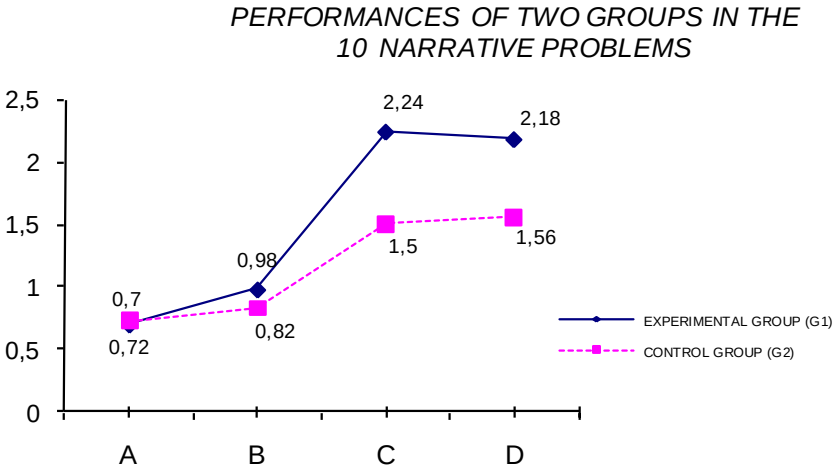
Additionally, during the last measurement a hundred and twenty nine (129) children (61 boys, 68 girls) from other 11 kindergarten classes, aged from 5*6 of (5 years and 6 months) to 6*6 (average=6, 06, while 63,6% of children were older than 6) were taught the approach of number concept as this is described in the official curriculum of kindergarten.

The measurement scale of the children's performances on all problems was from 0 to 4. The statistical package (SPSS) was used for the hypothesis control and the inductive statistical data analysis.

BRIEF PRESENTATION OF RESULTS

Gronbach's alpha factor of reliability was applied to the total of all narrative problems in our research; all reliability factors were found high: A: 0,93, B: 0,94, C: 0,95, D: 0,95.

The following diagram depicts the comparative development of average performances of both groups in the 10 narrative problems at phases A, B, C and D. Moreover, the parallel line control of the diachronic development of both groups is carried out. The continuous line refers to the experimental group and the pointed line refers to the control group.



The study of results of the above diagram on the narrative problems allows us to formulate the following observations:

Both during the implementation of the programme (A-B-C) and after its completion (D), the medium performances of the experimental group are higher than the performances of the control group, as far as these particular problems are concerned. In other words, the experimental group exhibits more progress. Comparing the diachronic development of the performances of both groups in the narrative problems (hypothesis control of both parallel lines), it is proved that the interaction of diachronic evolution within the subjects with the group type is statistically important (GLM Repeated Measures-Sphericity Assumed: $F=30,81$, $p<0,01$). If these differences are connected with the diagram results, we conclude that in the specific problem solving, the experimental group surpasses the control group throughout the whole duration of the experimentation, as far as their progress is concerned. The control group presents an ameliorative tendency in their performances from one testing to the next one. Likewise, we get the same result with the experimental group with a sole exception: from test C to test D there is a small decline in the solution of the 10 problems (from 2,24 to 2,18). This phenomenon should be attributed to the four-month period without any teaching between test C in June and test D in October.

The familiarization of preschool children with number analysis activities encouraged the development of problem solving abilities, although the children had not been previously taught the same problems, which they were asked. The results of this research show that the children were capable of applying the knowledge of analysis to other aspects of the number concepts (narrative problems of addition and subtraction). Finally, these results are long-lasting, as this was proved with the repetition of the test when the children were in the first class of elementary school. This superiority of the experimental group (G1) against the control group (G2) should be attributed to the unique specific difference between both groups:

particularly, the special instructive intervention that builds on the children's previous numerical experiences, the use of familiar and experientially attractive teaching aids and also the partition-composition of numbers.

The following table presents a comparative picture of the diachronic evolution in the rates of success of both groups in each of the narrative problems.

Diachronic evolution of the total success rates of both groups in the narrative problems

Description of the narrative problems	*GR.*	*A*	*B*	*C*	*D*
1. Increase with unknown the final situation (2+6=x)	G_1 G_2	8,5% 9,3%	14,0% 11,6%	72,9% 38,8%	70,5% 41,1%
2. Increase with unknown the final situation (3+4=x)	G_1 G_2	12,4% 13,2%	24,0% 15,5%	78,3% 40,3%	79,1% 41,9%
3. Decrease with unknown the final situation (8–6=x)	G_1 G_2	11,6% 10,1%	23,3% 15,5%	77,5% 35,7%	72,1% 40,3%
4. Decrease with unknown the final situation (8–4=x)	G_1 G_2	10,9% 12,4%	24,8% 16,3%	79,8% 46,5%	75,2% 48,1%
5. Increase with unknown the added quantity (3+x=7)	G_1 G_2	7,8% 10,1%	17,8% 12,4%	53,5% 32,6%	52,7% 34,1%
6. Increase with unknown the added quantity (2+x=8)	G_1 G_2	6,2% 7,0%	14,7% 9,3%	49,6% 31,8%	49,6% 33,3%
7. Finding of the whole (2+4=x)	G_1 G_2	11,6% 12,4%	23,3% 15,5%	82,9% 45,0%	76,7% 48,8%
8. Finding of the whole (5+2=x)	G_1 G_2	10,9% 10,1%	19,4% 13,2%	78,3% 43,4%	73,6% 46,5%
9. Finding of the part (7=4+x)	G_1 G_2	10,1% 10,1%	23,3% 16,3%	67,4% 35,7%	62,8% 34,9%
10. Finding of the part (8=6+x)	G_1 G_2	9,3% 9,3%	21,7% 15,5%	63,6% 34,1%	61,2% 35,7%

The results of the research brought to light the abilities of preschool children in the solution of problems, their difficulties and also the rich variety of their strategies. Below we will attempt to analyze these subjects.

DISCUSSION

The table above provides information on the rates of success of both groups in each of the 10 narrative problems and it allows us to formulate the following observations:

A first finding is that in each of the measurements (B, C, D) the success rates among the children of the experimental group are higher than the respective rates of the control group for all problems. The children were proved to be capable of solving a wide spectrum of problems with narrative stories. The context of the problems represented familiar situations that could reveal the children's informal

knowledge and therefore these problems were meaningful to them. Their performances were high. In particular, when the children started attending the first class of elementary school (in October), the percentage of success ranged from 49,6% to 82,9% in the experimental group and from 31,8% to 48,8% in the control group. The results of the experimental group are impressive, if we take into account that urban layers are not represented in the sample due to the fact that the social composition of the population in the Corinthian prefecture includes suburban, rural and working layers.

A second finding concerns the effect of the semantic structure in the difficulty of problems: problems that differ in the semantic structure, but are solved with the same numerical operation, differ in the degree of difficulty. This conclusion agrees with the findings of other researches (Carpenter & Moser, 1983 – Riley et *all*, 1983). Despite the fact that each problem constitutes a separate case, these problems can be divided into two categories:

- the easy problems (of increase or decrease with the final situation as the unknown quantity and the finding of the whole), in which the children of the experimental group presented rates of success from 70,5 to 82,9% and
- the difficult problems (the second quantity, finding of part), in which the rates of success for the experimental group were 49,6% to 67,4%.

The children comprehended easy problems of change (increase or decrease) with unknown the final situation and problems of combination with the whole as an unknown quantity. Children meet problems with increase or decrease at an early stage in their social environment. In a like manner, they face problems of combination which are reported in certain familiar classes that are enclosed in a total class (the kittens and the doggies are animals, the boys and the girls are children). The vocabulary and the inherent conceptual relations were comprehended by the infants, although the relations of inclusion are difficult for them.

The children had difficulties with the problems of the second category (missing added problems, finding of part). The major difficulties mainly related to the comprehension of these problems. In addition, the children found it difficult to represent the data with their fingers. The relationship between 'part'-'whole', which is inherent in the semantic structure of these problems, is of different nature. However, the children's performances are high, mostly due to the application of a programme of Mathematic activities that placed emphasis on the numbers' relationships of decomposition-composition. Besides, most problems were familiar to the children and they could mobilize informal strategies that they had formed in their social environment. Finally, they made progress passing from solutions of problems with direct material representations (by using their fingers) to the use of strategies concerning the analysis and recombining of numbers (Sophian & McCorgray 1994).

However, from the presentation of the findings, we can conclude that in narrative problems, the children of the experimental group excel not only in the

total rate of success, but also in the type of strategies. The strategies selected by the children of the experimental group are more mature than the corresponding ones of the control group. This can be seen in the increased percentages in all strategies and also in the big superiority of the experimental group in the strategies of analysis on fingers (G1: 23,10%, G2: 6,13%), that recommend the intermediary pre-mental stage for the passage from the processes with objects to the strategies without objects. The supremacy of the experimental group (G1) is explicit (G1: 21,40%, G2: 16,68%) in every test that concerned mental strategies. The same thing happened with the counting strategies with fingers (G1: 22,88%, G2: 17,68%). We can conclude that the familiarization of the children with the analysis and recombining of the finger patterns help in the progressive formation of the number concepts. It would be interesting to try this hypothesis with children of the first class of primary school mainly on the ascending and descending surpassing of the ten. However, each child does not show consistency concerning the use of strategies. The strategies differ both between problems and also within the same problem. The children frequently change their strategy during their effort to give an answer to the problem. They mobilize various strategies from the ones they know or they re-establish their familiar strategies and construct new ones.

CONCLUSIONS

Summarizing the above findings, we conclude that the children in the experimental group, who had been taught the number concept according to the approach of analysis, demonstrated a wide and composite capacity for problem solving. Our findings show that the children of the experimental group exceeded the level of abilities determined by the curriculum aimed for the first term of the first class of primary school. Nevertheless, the children of the control group made important progress, although the comparison between both groups shows a statistically important precedence of the experimental group. All children that took part in the research started primary school, possessing more knowledge than usual.

First of all, the above findings present an opposition to the findings of Piaget. The emergence of the complicated knowledge of children about the resolution of problems depends on the rich learning experiences which they acquired from the teaching program. The approach method of number with emphasis on the number analysis that we applied in the specific kindergartens came in accordance with the preexistent knowledge of children. We suppose that children possessed a latent set of principles for the additive analysis and composition before they could successfully face Piaget's experiment on class inclusion. In addition, they disposed a fertile prerequisite of numerical conservation of numbers before being able to answer the classic conservation experiment of Piaget. This knowledge was limited to the numerical conservation and the numerical inclusion and it was not generally extended in the conservation and the reasonable inclusion of classes. All preexistent knowledge of children is useful for the later conceptual development.

According to Resnick, these are pre-quantitative logic schemes and constitute the foundation of mathematic development of children (Resnick, 1983).

The instructive approach in the kindergarten according to the official Curriculum presents the following characteristics: unilateral persistence in the pre-requisite logic concepts (classifications serial-ordering), depreciation of the importance of numbers analysis, not open approach, absence of use of fingers, dogmatic and mechanic transport of psychological experiments in the teaching.

The biggest part of the syllabus for mathematics in pre-school includes skills that directly or indirectly depend on the comprehension of relations of numbers analysis. The results of this study show that the instructive intervention, which stresses the analysis of numbers, can assist the development of number concepts and the abilities associated with them. It seems that the differences found between both groups are connected with the difference of emphasis between both educational methods (the set of activities of reflection and communication and the usual syllabus of mathematics of kindergarten).

Consequently, the familiarization of infants with analysis activities encouraged their development of abilities to resolve narrative problems of addition and subtraction. The results of this research show that the children are capable of applying the knowledge of analysis to various aspects of number concepts. A similar teaching approach can provide the suitable base for the interpretation of the small and the big numbers and calculations. In general, the instructive familiarization with numbers analysis activities supported the development of the basic number concepts, i.e. addition and subtraction. These relations recommend an important foundation that stresses a lot of mathematic concepts that young children develop. What is more, these results are long-lasting, as it was proved by the test given to the pupils in the first class of primary school.

With which methods and with which means will a wider comprehension of mathematics be achieved? How will the children invent suitable strategies in order to solve new problems?

Our present work attempts to give certain initial answers to the above questions. Neither Piaget's view (underestimation of young children) nor the approach of modern constructivists (emphasis on the numeration and the construction of abstract units) were adequate for us. Naturally, we are not satisfied with behaviourism, either. Despite its importance, the numerical analysis of numbers is inadequately presented in Piaget's theory. His interest was focused mainly on logic inclusion and preservation. The latter term is the guarantee for the real comprehension of number. Children should understand that the partition and the reunification, as well as the comprehension of addition and subtraction are acquired at the age of 7 years during the process of maturation (stage of concrete thought).

The analysis is also absent from the teaching experiments of constructivism theory. The 'part-whole' relations are examined in the context of sequence of numbers (Steffe, 2004). Moreover, they do not explain which items of knowledge should be acquired by children during the pre-mental levels in order to change

progressively their strategies and reach the abstract stage of enumeration and the invention of thought strategies (mental calculation). Finally, they do not suggest activities with the use of teaching aids that could function as "cognitive bridges" facilitating and preparing these constructions. Analysis is absent from behaviorism, in which the priority is given to the mechanical memorization of sums and differences without conceptual comprehension.

In our approach we considered that the progressive mathematization should be connected with the mathematics of school. Therefore, we were released by the experiment of reservation and inclusion. The acquisition of variety of experiences with the analyses of numbers is judged essential. The use of each number in a lot of different cases contributes to the conquest of the number concept. With the present work we propose a method of approach towards numerical notions in pre school education kindergarten, which is based on the analysis of the number itself. A beam of instructive activities that aimed at the numerical development of children was implemented.

The high performances of the infants, who have been initiated in the world of numbers with this method, reveal its effectiveness. However, we cannot conclude that the quality of the number approach is better, judging only from the difference of learning performances. This is one aspect of the problem. The establishment of this approach and the content of the group of instructive activities are more important (Kosyvas, 2001).

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