ЮБИЛЕЙНА НАУЧНА СЕСИЯ – 30 години ФМИ
ПУ "Паисий Хилендарски", Пловдив, 3-4.11.2000

PRIMARY MATHEMATICAL MODEL OF THE HEAT EXCHANGE FOR THE RF-EXCITED GAS DISCHARGE IN HE-CD LASER SYSTEM. Part I – CONSTRUCTION OF THE MODEL

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This paper is devoted to the phenomena of thermal instability of the gas discharge, which accompanied the laser technology since its invention and never be satisfactory explained. An attempt to construct a mathematical model, describing the heat conductivity of the radio frequency (RF) gas discharge in He-Cd laser is proposed. The model is based on the classical 2D-heat conduction equation for the gas temperature, subject to the boundary value problem in a cross-section of the laser tube. The necessary parameters are determined. The model can be used to investigate the reaction of the surroundings with respect to the increase of the applied electric power at maintenance of a constant optimal gas temperature.[1]

Introduction.

The thermal instability and overheating of the plasma discharge in different devices in laser technology are well known, but not well investigated. Some notes and particular results are given in [1] for the case of radio frequency (RF) gas discharge and in [2] for dc-discharge.

The increase of the applied power (i.e. RF power delivered to the discharge) in axial RF-discharge lasers has its natural limits. When the gas temperature T_g exceeds certain critical rates, there arise thermoionization phenomena involving gas discharge deterioration. The high current density leads to decrease of the amount of excited atoms in upper states. That results in laser generation collapse or in its worse quality. All these processes depend also on the heat exchange between the boundaries of a working system and the surroundings. The investigation of the mentioned above must allow us to find engineering solutions for intensification of the heat transfer in order to increase the gas discharge electric power at a constant optimal laser generation temperature. For this reason we constructed an appropriate mathematical model and carried out numerical calculations for a real case of laser.

We will consider a RF excited He-Cd laser, having two external longitudinal electrodes and operating at a frequency of 13.56 MHz. The laser design, geometry and data for gas pressure and electric power are given in [3] (see also Fig.1). The RF discharge excited the He-Cd mixture inside an AL_2O_3 capillary tube, inserted into quartzous tube. The voltage is aplied across the two outer electrodes, wrapped around the external tube. The cadmium is placed in a supplementary reservoir and penetrates in the gas discharge after certain pre-heating in an oven. The helium pressure rate is 30 mbar and the initial applied electric power was taken to be 360W. We are unable to know the exact values of the gas discharge into the laser tube, but in principle, it can be characterized as a so called γ- discharge, because of it's high current density [2].

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[1] AMS Subject Classification. 81V80, 78A60

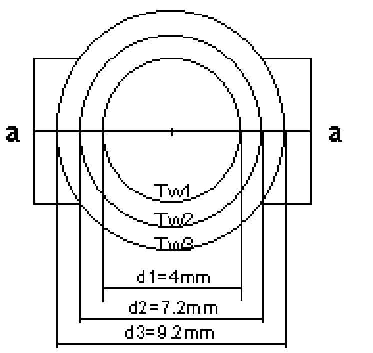

Fig.1. Geometry of the cross-section of the He-Cd laser tube.

In this paper we study the connection between the applied electric power and heat transfer reaction in the surroundings. The treatment takes into account the effect of two types of heat transfer: by convection and by radiation.

Assumptions and description of the mathematical model.

The investigation on RF discharge [1], [2] shows the availability of quasi steady state of the positive ions, which determine certain fixed component of electrical field. Hence, one can consider a heat transfer model in quasi-stationary case, i.e. independently with respect to the time variable. The electrons oscillate around the quasi steady-state ions, but the availability of a noncompensate positive discharge around the two electrodes is also possible. In this case, for simplicity, we shall assume the electrons in our model being in certain average steady state.

The distribution of the gas temperature T_g for a RF He-Cd laser, under operating conditions satisfies the steady-state heat-conduction equation of the form:

(I.1) $$\nabla\lambda_g\nabla T_g = -jE$$

where λ_g is the heat-conductivity coefficient of the gas (here helium), j – current density, E – the intensity of the electric field. The solution of equation (I.1) is seeking in two-dimensional domain, describing a cross-section of the laser tube (Fig.1).

For the boundaries we have [5]:

(I.2) $$Q=\alpha F(T_{w3}-T_f) + F\varepsilon_0 c(T_{w3}/100)^4, \quad \text{or} \quad Q=Q_1+Q_2 ,$$

(I.3) $$\lambda_{w1}\frac{\partial T_{w1}}{\partial n} = \lambda_{w2}\frac{\partial T_{w2}}{\partial n} ,$$

where Q is the heat flow, α - the heat transfer coefficient, F – outside surface area, T_{w1},T_{w2} and T_{w3} – the temperatures on the corresponding surfaces of the walls, T_f – the temperature of the surroundings, ε_0 – integral emmisivity of black body, c= 5.67 W/(m^2K^4) – coefficient of radiation of the black body and λ_{w1}, λ_{w2} - thermal conductivities of the walls. The boundary condition (I.2) describes the heat-transfer law between the outside surface of the tube to the surroundings. The right hand side is composed of two terms $Q_1$and Q_2. The first of them means the Newton's law of heat transfer by convection. The second term expresses the Stefan-Boltzmann law of the heat transfer by radiation. The condition (I.3) includes also the heat exchange between the two contacting surfaces of the composite wall (see Fig.1).

Usually the thermal conductivity of the gas λ_g is given in the form $\lambda_g = \lambda_0 T_g^m$, where λ_0 is a constant. For helium, by using the data from [6] for $T_g \in [600, 1000]K$ and the least square method we obtain m=0.7016 at $\lambda_0 = 2.15.10^{-5}\ W/(sm.K^{1.7016})$. If $U = T_g^m$ the equation (I.1) is reduced to the Poisson equation:

(I.4) $$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}=-\varphi(x,y) \quad ,$$

where the right-hand side $\varphi(x,y)$ is given. Let us denote by $\eta=Q_1/Q$ the part of the common power which is transferred by convection. Now from the condition (I.2):

(I.5) $Q=\alpha F(T_{w3} - T_f)\eta$, $\eta=(Q-F\varepsilon_0 c(T_{w3}/100)^4)/Q$.

Let introduce the linear density of the heat flow Q by:

(I.6) $q_l= Q_1/l$,

where l is the length of the tube. Despite the fact, that the cylindrical wall of the laser tube is composed of two materials, (i.e. of two layers) the heat flow Q_1, generated by the gas discharge, will be the same while the heat is transferred through the wall. Hence, the boundary condition (I.3) can be divided into two parts [5]:

(I.7) $T_{w2}=T_{w3}+(q_l/2\pi\lambda_{w2})\ln(d_3/d_2)$, $T_{w1}=T_{w2}+(q_l/2\pi\lambda_{w1})\ln(d_2/d_1)$.

Then, by applying (I.6) and (I.7) the boundary conditions (I.2),(I.3) can by written as:

(I.8) $$T_{w3} = T_f + \frac{q_l \eta}{\pi d_3 \alpha} \ , \quad T_{w2} = T_{w3} + \frac{q_l}{2\pi\lambda_{w2}} \ln\frac{d_3}{d_2} \ , \quad T_{w1} = T_{w2} + \frac{q_l}{2\pi\lambda_{w1}} \ln\frac{d_2}{d_1} \ ,$$

where $d_{1,}d_2$ and d_3 are respectively the diameters of the walls (see Fig.1).

Thus from the problem (I.1-3) we obtain a heat exchange mathematical model, describing by the Poisson equation (I.4) and the boundary conditions of the first kind, given by (I.8).

Determination of the Necessary Parameters.

In order to solve numerically the mathematical model (I.4)- (I.8) we need to find all the parameters taking part in the right hand sides of these equations.

1) The values of intensity E were calculated by solving the Poisson equation as it was presented in [7],[8].

2) To find the current density j we use the expression $j=en_e v_d$, where n_e is the electron concentration and the draft velocity v_d was approximated by the data from [6] as a function in the form $v_d=f(E/p)$, p - being the gas pressure.

3) To find the value of η (see (I.5)) let T_{w3}=660K, ε_0=0.86 [6] and area of cooling F=0.023m^2. Then from the Stefan-Boltzmann law we obtain a radiation power of 213W. For the heat transfer by convection it remains 147W, so η=0.408 of the common power. Therefore, the primary way of heat transfer from the laser tube to the surroundings is by radiation. The analogous results are obtained in [2] for dc-discharge.

4) The value of the heat transfer coefficient α depends on the way of the heat exchange between the body and the surroundings. The heat transfer by convection is due to the fluid moving around the outside surface of the tube, while the fluid takes away some heat from the tube. We shall consider two types of convection: natural and forced convection.

4.1) Natural convection. For all ways of convection we can use the Nusselt number [5]:

(I.9) $\mathrm{Nu} = l\alpha/\lambda$,

where l is the characteristic linear dimension of the body, i.e. it's external diameter and λ - thermal conductivity of the fluid. From (I.9) we have:

(I.10) $\alpha = \lambda\, Nu / l$.

In the case of a free convection the Grashof number is [5]:

(I.11) $\mathrm{Gr} = g\,\beta\,l^3\,\Delta T / \upsilon^2$,

where g is the gravitational acceleration, β - coefficient of cubical heat expansion of the gas, $\Delta T=T_{w3}-T_f$ -temperature difference and ν - kinetic viscosity of the fluid. For the horizontal tubes with natural air convection if $700<Gr<7.10^7$ the next equality holds [10]:

(I.12) $Nu=0.46Gr^{0.25}$.

Let in our model we take the air temperature T_f=300K and $\Delta T=T_{w3}-T_f$=360K, the external diameter $l=d_3$=9.2mm, $\beta=3.41.10^{-3}K^{-1}$, λ=0.0251W/(m.K), $\nu=15.7.10^{-6}m^2/s$. From (I.9), (I.11), (I.12) we find $Gr=3.47.10^4$, Nu=6.28 and $\alpha=17.61W/(m^2\,K)$.

4.2) Forced convection. To determine the heat transfer coefficient α in the case of a forced convection we use the Reinold's number

(I.13) $\mathrm{Re} = lv/\upsilon$,

with v - the velocity of the moving fluid, 40<Re<4000 [10].

For the horizontal tubes at a forced cooling in air from [10]:

(I.14) $Nu=0.615\ Re^{0.466}$.

Hence, by using (I.9),(I.13) and (I.14) we calculate

(I.15) $\alpha=32.6\ v^{0.466}$.

Acknowledgment.

This work was supported by the Commission of the European Community under Copernicus Program CIPA-CT93-0219.

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