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GATEAUX DIFFERENTIABILITY OF BUMP FUNCTIONS IN SEPARABLE BANACH SPACES

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It is shown that in large class of separable Banach spaces with an unconditional basis there is no 2-times Gateaux differentiable bump functions. This result is applied in Orlicz and Lorentz sequence spaces. A condition for nonexistence of 2-times Gateaux differentiable bump functions in Orlicz and Lorentz sequence spaces is found.

Theorem 1. Let ℓ_m be an Orlicz sequence space with M satisfying the Δ_2-conditions at 0. Let $\omega : R^+ \to R^+$ be such that

$$\sup_{u,v\in(0,1]} \frac{M(uv)}{\omega(u)M(v)} = \infty.$$

Then there exists no bump $b \in G_{\omega,1}(\ell_M)$.

Theorem 2. Let be given a Lorentz space $d(w, p)$ and $\omega : R^+ \to R^+$ with $\omega(t) = \mu(t)t^p$, where $\liminf_{t\to 0} \mu(t) = 0$. Then there exists no bump $b \in G_{\omega,1}(d(w, p))$.

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