TECHNIQUES FOR ENHANCING THE DEVELOPMENTAL FUNCTION OF MATHEMATICAL TASKS

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ABSTRACT

The issue of enhancing the developmental component of school mathematical education is considered. The techniques for intensification of the developmental function of the mathematical tasks are given. Application of the proposed techniques at the lessons of mathematics in the 5th and 6th grades encourages the development of mathematical thinking of pupils. The peculiarities of the techniques are described, the examples of use are shown.

Key words: teaching mathematics, developmental task, developmental function, mathematical thinking, the 5th and 6th grades.

Mathematics is one of the most essential basic educational subjects aimed at building up the all-round educated personality. In modern society mathematical knowledge is not only thought of as an end in itself but considered to be a means of personality development. Therefore, one of the main tasks of improvement in Ukrainian school mathematical education is to strengthen its developmental component. In order to complete this it is necessary to introduce such methods, organizational forms and educational means which encourage the personality development of the pupils, in particular, their mathematical thinking. Since mathematical thinking gets its most intensive progress during the process of solution of various mathematical tasks, we face an important issue: is to create a system of tasks of a certain orientation and teaching the pupils how to solve them. According to modern demands they should be defined as developmental tasks.

In the literature on methodology one might find different classifications of such tasks. We are guided by classification of N.V. Metelskiy (Metelskiy, 1982), and distinguish between: cognitive (providing some new knowledge), training (whose main purpose is to perfect skills), and developmental (requiring creative thinking) tasks.

The task can be referred to as a developmental one provided the pupils, while solving them, learn to compare familiar and unfamiliar facts, combine and think, generalize the obtained solutions, draw the conclusions. These are the tasks, which way of solution is not evident, and one has to apply heuristics in order to find it.

The main purpose of use of the developmental tasks is elaboration of mathematical thinking. Quantity, sequence and place of mathematical developmental tasks in the system of lesson objects are established by a teacher according to the main object of certain lesson. It is necessary for each topic of the mathematical course for the 5^{th} and 6^{th} grades to contain not less than one third of developmental tasks of total quantity.

Till recently the majority of school tasks have been training ones, aimed at forming conscious and profound skills and abilities of applying mathematical knowledge. Calculation tasks prevailed, for calculation skills have been considered the basic ones in the 5^{th} and 6^{th} grades. Ukrainian school education reforms, observed these days, provide some positive changes. A new program in mathematics has been adopted; new course books for 5^{th} and 6^{th} grades have appeared. When analyzing the tasks in these course books (Bevz, 2005, 2006, Merzlyak, 2005, 2006) one can conclude, that the quantity of developmental tasks has increased. However, the ratio between them and the tasks of other types, to our mind, remains inadequate.

Thus, we face a necessity to supplement the existing set, suggested by a course book, with additional developmental tasks. This problem can be fulfilled by a teacher in three ways: to select the developmental tasks from supplementary materials; to compose the tasks by himself, taking into account the specificity of a given class; to intensify the developmental function of the tasks from the current course book.

We suggest the techniques that allow enhancing the developmental function of the tasks taken from the course books in mathematics for the 5^{th} and 6^{th} grades in this paper.

Techniques for enhancing the developmental function of mathematical tasks. To them we refer (Metelskiyi, 1982):

- construction of different writing of ask situation;
- broadening the set of questions on the task situation;
- solving the task in different ways;
- substitution of numeral values by letter symbols;
- composing a task.

Employment of the mentioned techniques in the work with the tasks allows us, firstly, to promote the purposeful development of mathematical thinking (namely, such its components as analysis, generalization, abstracting, planning, and reflection); and secondly, to raise pupils' activity during the lesson. Let's examine some peculiarities of the above techniques arising during their application to tasks solving in the course of mathematics for 5^{th} and 6^{th} grades.

1. Construction of different writing of ask situation. This technique implies creation of various forms of the written statement of a given task: figure, schematic notes, graphical scheme, flow-graph, table, diagram, etc. Such procedure stimulates

the interest to the task, trains to apply a profound analysis of the situation, allows pupils to present it clearly, makes the expected result demonstrable, and conveys the information in concise form.

2. Broadening the set of questions on the task situation. This means constructing various questions for the task situation. The questions can be of two types: those requiring answer during the process of solution, and those, which answers state some new tasks. It should be noted, that the questions should be diverse, e.g.: "How many…", "Calculate…", "Compare…", "Find the regularity…", "Is it possible that…", "If we change… then…" and others.

According to N.A. Tarasenkova (Tarasenkova, 2002), the ability to formulate questions in many cases serves not only for expanding a certain way of solution but also for the choice of the way itself. The variety of questions increases the interest to the task, because pupils start to understand better its content and find the dependences between all values.

3. Solving the task in different ways. Applying this technique, a teacher offers the pupils to solve the task in different ways, choose among them the most efficient among them, and substantiate their choice. In the 5^{th} and 6^{th} grades it can be widely used, while solving text tasks in arithmetic and algebraic ways, and when carrying out calculations.

4. Substitution of numeral values by letter symbols. The pupils are offered to read a statement of a task, substitute all numerical values by letters, and solve the task using ordinary reasoning. Note, that this is the technique that develops the ability of abstracting and generalization most effectively.

5. Composing a task. Application of this technique implies two possibilities: a teacher composes tasks by himself, based on tasks given in a course book, or asks the pupils to do this. Furthermore, the tasks can be similar to the given one in the statement, formulation of requirements, or inverse to it. It is advisable that the tasks stick to different content: calculation, movement, cooperation, geometrical and others. In solution of the obtained tasks one should emphasize those with the way of solution analogous to the original task.

In the performed investigation we have found, that the techniques indicated above can be used both separately and in the combined manner. This is depends on the goal of the lesson, the level of the class in general and the level of each individual.

Let's consider an example of how a combination of techniques can help to enhance the developmental function of the training task taken from a school course book (Bevz, 2005).

Task

Two cyclists simultaneously started moving towards each other from two villages. The distance between the villages is 60 km. They met in 2 hours. The first cyclist was traveling at a speed of 14 km/h. Find the speed of the second cyclist.

After analyzing the task situation, technique 1 implies that the pupils are able to write down a short statement using a table or graphical scheme. After that a teacher proposes to solve the task in different ways (technique 3): arithmetic and algebraic ones.

On the next stage it is appropriate to formulate additional questions (technique 2).

- Which cyclist was traveling at higher speed? By how many?
- Which cyclist has traveled a longer distance to the place of meeting? By how many? Explain, why.
- What was the distance between the cyclists an hour after they started moving?
- What will the distance between the cyclists be three hours after they start moving? Explain the obtained answer.
- Compare the distances between the cyclists for one and three hours after they start moving. Explain the obtained answer.

To apply technique 5 we can offer the pupils to consider the movement of cyclists in opposite directions, or in the same direction, instead towards each other.

Reflection of educational activity, done by the pupils is a highly essential component of work at a task is. The questions like "What do you think?", "Why do you think so?" train the pupils to substantiate all their assumptions and conclusions while reasoning.

It is worth noting, that use of the combined techniques increases the developmental function of the task to a great extent. Search for different ways of solution, formulation of questions on the task situation, and solution of the composed tasks makes the pupils conscious participants of the educational process, develops the components of mathematical thinking.

It has been determined, that the shown techniques can be applied to many tasks of the mathematical course for 5^{th} and 6^{th} grades. The issues of tasks selection, periodicity of application of the techniques, and search for other ways of enhancing the developmental function of the tasks need further investigation.

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