ПЛОВДИВСКИ УНИВЕРСИТЕТ "ПАИСИЙ ХИЛЕНДАРСКИ",
БЪЛГАРИЯ
НАУЧНИ ТРУДОВЕ, ТОМ 33, КН. 3, 2001 – МАТЕМАТИКА
PLOVDIV UNIVERSITY "PAISSII HILENDARSKI", BULGARIA
SCIENTIFIC WORKS, VOL. 33, BOOK 3, 2001 – MATHEMATICS

ON THE CURVATURE PROPERTIES OF ALMOST CONTACT B-METRIC HYPERSURFACES OF KAEHLERIAN MANIFOLDS WITH B-METRIC

Mancho Hristov Manev

Abstract. A type of almost contact B-metric hypersurfaces of a Kaehlerian manifold with B-metric is considered. There are characrerized the curvature tensors and the special sectional curvatures. There are considered the corresponding curvature properties in the case of the main class of these hypersurfaces.

***Mathematics Subject Classifications* 2000:** Primary 53C15, 53C40; Secondary 32Q60, 53C50.

***Key words*:** hypersurface, curvature, almost contact manifold.

Introduction

The Kaehlerian manifolds with B-metric have been introduced in [**?**]. These manifolds form the special class $\mathcal{W}_0$ in the classification of the almost complex manifolds with B-metric, given in [**?**]. This most important class is contained in each of the basic classes in the mentioned classification.

The natural analogue of the almost complex manifolds with B-metric in the odd dimensional case are the almost contact B-metric manifolds, classified in [**?**].

In [**?**] we constructed two types of hypersurfaces of an almost complex manifold with B-metric as almost contact B-metric manifolds, and there we determined the class of these hypersurfaces of a $\mathcal{W}_0$-manifold.

An important problem in the differential geometry of the Kaehlerian manifolds with B-metric is the studying of the manifolds with constant totally real sectional curvatures [**?**].

In this paper we study some curvature properties of hypersurfaces of second type of a Kaehlerian manifolds with B-metric and particularly curvature properties of nondegenerate special sections.

1. Preliminaries

1.1. **Notes on almost complex manifolds with B-metric.**

Let (M', J, g') be a $2n'$-dimensional almost complex manifold with B-metric, i.e. J is an almost complex structure and g' is a metric on M' such that:

$$J^2X = -X, \qquad g'(JX, JY) = -g'(X, Y) \tag{1.1}$$

for all vector fields X, Y on M' . The associated metric $\widetilde{g'}$ of the manifold is given by $\widetilde{g'}(X,Y) = g'(X,JY)$. Both metrics are necessarily of signature (n',n').

The tensor field F' of type (0,3) on M' is defined by $F'(X,Y,Z) = g'((\nabla'_X J)Y,Z)$, where ∇' is the Levi-Civita connection of g', and $X,Y,Z \in \mathfrak{X}(M')$ (the Lie algebra of the differentiable vector fields on M').

A classification with three basic classes of the almost complex manifolds with B-metric with respect to F' is given in [**?**]. In this paper, we shall consider only the class $\mathcal{W}_0 :\ F' = 0$ of the Kaehlerian manifolds with B-metric belonging to each of the basic classes. The complex structure J is parallel on every $\mathcal{W}_0$-manifold, i.e. $\nabla' J = 0$.

The curvature tensor field R' defined by

$$R'(X, Y)Z = \nabla'_X\nabla'_Y Z - \nabla'_Y\nabla'_X Z - \nabla'_{[X,Y]}Z$$

possess the property $R'(X, Y, Z, U) = -R'(X, Y, JZ, JU)$ on a $\mathcal{W}_0$-manifold. Using the first Bianchi's identity and the last property of R it follows $R'(X, JY, JZ, U) = -R'(X, Y, Z, U)$.

Therefore, the tensor field $\widetilde{R'} : \widetilde{R'}(X, Y, Z, U) = R'(X, Y, Z, JU)$ has the properties of a Kaehlerian curvature tensor and it is called an associated curvature tensor.

For every nondegenerate section α' in $T_{p'}M'$, $p' \in M'$, with a basis $\{x, y\}$ there are known the following sectional curvatures with respect to g [**?**]:

$k'(\alpha'; p') = k'(x, y) = \frac{R'(x,y,y,x)}{\pi_1'(x,y,y,x)}$ – the usual Riemannian sectional curvature;

$\widetilde{k}'(\alpha'; p') = \widetilde{k}'(x, y) = \frac{\widetilde{R}'(x,y,y,x)}{\pi_1'(x,y,y,x)}$ – an associated sectional curvature,

where $\pi_1'(x, y, y, x) = g'(x, x)g'(y, y) - [g'(x, y)]^2$.

The sectional curvatures of an arbitrary holomorphic section α' (i.e. $J\alpha' = \alpha'$) is zero on a Kaehlerian manifold with B-metric [**?**].

For totally real sections α'(i.e. $J\alpha' \perp \alpha'$) is proved the following

Theorem 1. [[**?**]]Let M' $(2n' \geq 4)$ be a Kaehlerian manifold with B-metric. M' is of constant totally real sectional curvatures $\nu'$and $\widetilde{\nu'}$, i.e. $k'(\alpha'; p') = \nu'(p')$, $\widetilde{k}'(\alpha'; p') = \widetilde{\nu'}(p')$ whenever α' is a nondegenerate totally real section in $T_{p'}M'$, $p' \in M'$, if and only if

$$R' = \nu' \left[\pi_1' - \pi_2'\right] + \widetilde{\nu'}\pi_3'.$$

Both functions $\nu'$and $\widetilde{\nu'}$are constant if $M'$is connected and $2n' \geq 6$.

The essential curvature-like tensors are defined by:

$$\begin{aligned}
\pi_1'(x, y, z, u) &= g'(y, z)g'(x, u) - g'(x, z)g'(y, u),\\
\pi_2'(x, y, z, u) &= g'(y, Jz)g'(x, Ju) - g'(x, Jz)g'(y, Ju),\\
\pi_3'(x, y, z, u) &= -g'(y, z)g'(x, Ju) + g'(x, z)g'(y, Ju)\\
&\quad - g'(y, Jz)g'(x, u) + g'(x, Jz)g'(y, u),
\end{aligned}$$

1.2. **Notes on almost contact manifolds with B-metric**

Let $(M, \varphi, \xi, \eta, g)$ be a $(2n+1)$-dimensional almost contact manifold with B-metric, i.e. (φ, ξ, η) is an almost contact structure determined by a tensor field φ of type $(1, 1)$, a vector field ξ and a 1-form η on M satisfying the conditions:

$$\varphi^2 X = -X + \eta(X)\xi, \qquad \eta(\xi) = 1, \tag{1.2}$$

and in addition the almost contact manifold (M, φ, ξ, η) admits a metric g such that

$$g(\varphi X, \varphi Y) = -g(X, Y) + \eta(X)\eta(Y), \tag{1.3}$$

where X,Y are arbitrary differentiable vector fields on M, i.e. $X,Y \in \mathfrak{X}(M)$[?].

There are valid the following immediate corollaries:

$$\eta\circ\varphi=0,\ \varphi\xi=0,\ \eta(X)=g(X,\xi),\ g(\varphi X,Y)=g(X,\varphi Y). \tag{1.4}$$

and φ is an endomorphism with rank $2n$.

The associated metric $\widetilde{g}$ given by $\widetilde{g}(X,Y)=g(X,\varphi Y)+\eta(X)\eta(Y)$ is a B-metric, too. Both metrics g and $\widetilde{g}$ are indefinite of signature $(n,n+1)$.

Further, X,Y,Z,U will stand for arbitrary differentiable vector fields on M and x,y,z,u – arbitrary vectors in tangential space T_pM to M at an arbitrary point p in M. The Levi-Civita connection of g will be denoted by ∇. The tensor field F of type (0,3) on M is defined by $F(X,Y,Z)=g((\nabla_X\varphi)Y,Z)$.

If $\{e_i,\xi\}$ $(i=1,2,\dots,2n)$ is a basis of T_pM and (g^{ij}) is the inverse matrix of (g_{ij}), then the following 1-forms are associated with F:

$$\theta(\cdot)=g^{ij}F(e_i,e_j,\cdot),\ \ \theta^*(\cdot)=g^{ij}F(e_i,\varphi e_j,\cdot),\ \ \omega(\cdot)=F(\xi,\xi,\cdot).$$

A classification of the almost contact manifolds with B-metric is given in [?], where eleven basic classes $\mathcal{F}_i$ are defined. In the present paper we consider the following classes:

$$\begin{aligned}
&\mathcal{F}_4: F(x,y,z)=-\frac{\theta(\xi)}{2n}\{g(\varphi x,\varphi y)\eta(z)+g(\varphi x,\varphi z)\eta(y)\};\\
&\mathcal{F}_5: F(x,y,z)=-\frac{\theta^*(\xi)}{2n}\{g(x,\varphi y)\eta(z)+g(x,\varphi z)\eta(y)\};\\
&\mathcal{F}_6: F(x,y,z)=F(x,y,\xi)\eta(z)+F(x,\xi,z)\eta(y),\\
&\qquad F(x,y,\xi)=F(y,x,\xi),\quad F(\varphi x,\varphi y,\xi)=-F(x,y,\xi)\\
&\qquad \theta(\xi)=\theta^*(\xi)=0;\\
&\mathcal{F}_8: F(x,y,z)=F(x,y,\xi)\eta(z)+F(x,\xi,z)\eta(y),\\
&\qquad F(x,y,\xi)=F(y,x,\xi),\quad F(\varphi x,\varphi y,\xi)=F(x,y,\xi).
\end{aligned}\tag{1.5}$$

The classes $\mathcal{F}_i\oplus\mathcal{F}_j$, etc., are defined in a natural way by the conditions of the basic classes. The special class $\mathcal{F}_0: F=0$ is contained in each of the defined classes. The $\mathcal{F}_i^0$-manifold is an $\mathcal{F}_i$-manifold $(i=1,4,5,11)$ with closed associated 1-forms.

The following tensors are essential curvature tensors on M:

$$
(1.6)\qquad
\begin{aligned}
\pi_1(x,y,z,u) &= g(y,z)g(x,u) - g(x,z)g(y,u),\\
\pi_2(x,y,z,u) &= g(y,\varphi z)g(x,\varphi u) - g(x,\varphi z)g(y,\varphi u),\\
\pi_3(x,y,z,u) &= -g(y,z)g(x,\varphi u) + g(x,z)g(y,\varphi u)\\
&\quad - g(y,\varphi z)g(x,u) + g(x,\varphi z)g(y,u),\\
\pi_4(x,y,z,u) &= \eta(y)\eta(z)g(x,u) - \eta(x)\eta(z)g(y,u)\\
&\quad + \eta(x)\eta(u)g(y,z) - \eta(y)\eta(u)g(x,z),\\
\pi_5(x,y,z,u) &= \eta(y)\eta(z)g(x,\varphi u) - \eta(x)\eta(z)g(y,\varphi u)\\
&\quad + \eta(x)\eta(u)g(y,\varphi z) - \eta(y)\eta(u)g(x,\varphi z).
\end{aligned}
$$

In [**?**] it is established that the tensors $\pi_1 - \pi_2 - \pi_4$ and $\pi_3 + \pi_5$ are Kaehlerian, i.e. they have the condition of a curvature-like tensor L:

$$
(1.7)\qquad L(X,Y,Z,U) = -L(X,Y,\varphi Z,\varphi U)
$$

Let R be the curvature tensor of ∇. The tensors R and $\widetilde{R} : \widetilde{R}(x,y,z,u) = R(x,y,z,\varphi u)$ are Kaehlerian on every $\mathcal{F}_0$-manifold.

There are known the following sectional curvatures with respect to g and R for every nondegenerate section α in T_pM with a basis $\{x,y\}$:

$$
k(\alpha;p) = k(x,y) = \frac{R(x,y,y,x)}{\pi_1(x,y,y,x)},\quad \widetilde{k}(\alpha;p) = \widetilde{k}(x,y) = \frac{\widetilde{R}(x,y,y,x)}{\pi_1(x,y,y,x)}.
$$

In [**?**] are introduced the special sections in T_pM: a ξ-section (e.g. $\{\xi,x\}$), a φ-holomorphic section (i.e. $\alpha = \varphi\alpha$) and a totally real section (i.e. $\alpha \perp \varphi\alpha$).

The canonical curvature tensor K is introduced in [**?**]. The tensor K is a curvature tensor with respect to the canonical connection D defined by

$$
(1.8)\qquad D_XY = \nabla_XY + \frac{1}{2}\left\{(\nabla_X\varphi)\varphi Y + (\nabla_X\eta)Y.\xi\right\} - \eta(Y)\nabla_X\xi.
$$

The connection D is a natural connection, i.e. the structural tensors are parallel with respect to D. Let us note that the tensor K out of $\mathcal{F}_0$ has the properties of R in $\mathcal{F}_0$.

We recall the following theorems:

Theorem 2. [[?]] Let $(M, \varphi, \xi, \eta, g)$ $(\dim M \geq 5)$ be an $\mathcal{F}_0$-manifold. M is of constant totally real sectional curvatures ν and $\widetilde{\nu}$, i.e. $k(\alpha; p) = \nu(p)$, $\widetilde{k}(\alpha; p) = \widetilde{\nu}(p)$, whenever α is a nondegenerate totally real orthogonal to ξ section in T_pM, $p \in M$, iff

$$R = \nu \left[\pi_1 \circ \varphi - \pi_2\right] + \widetilde{\nu}\pi_3 \circ \varphi.$$

Both functions ν and $\widetilde{\nu}$ are constant if M is connected and $\dim M \geq 7$.

Theorem 3. [[?]] Let $(M, \varphi, \xi, \eta, g)$ $(\dim M \geq 5)$ be an $\mathcal{F}_i^0$-manifold $(i = 1, 4, 5, 11)$. The manifold M is of constant totally real sectional curvatures ν and $\widetilde{\nu}$ of the curvature tensor K, i.e. $k(K)(\alpha; p) = \nu(K)(p)$, $\widetilde{k}(K)(\alpha; p) = \widetilde{\nu}(K)(p)$, whenever α is a nondegenerate totally real orthogonal to ξ section in T_pM, $p \in M$, iff

$$K = \nu(K) \left[\pi_1 \circ \varphi - \pi_2\right] + \widetilde{\nu}(K)\pi_3 \circ \varphi.$$

If M is connected and $\dim M \geq 7$ then functions $\nu(K)$ and $\widetilde{\nu}(K)$ satisfy the following conditions:

$a)$ for $i = 1$ $\quad d\nu = \frac{1}{2n}\left[\nu.\theta^* \circ \varphi - \widetilde{\nu}.\theta\right]$, $\quad d\widetilde{\nu} = \frac{1}{2n}\left[\nu.\theta + \widetilde{\nu}.\theta^* \circ \varphi\right]$;
$b)$ for $i = 4$ $\quad d\nu = -\frac{1}{n}\theta(\xi)\widetilde{\nu}\eta$, $\quad d\widetilde{\nu} = \frac{1}{n}\theta(\xi)\nu\eta$;
$c)$ for $i = 5$ $\quad d\nu = -\frac{1}{n}\theta^*(\xi)\nu\eta$, $\quad d\widetilde{\nu} = -\frac{1}{n}\theta^*(\xi)\widetilde{\nu}\eta$;
$d)$ for $i = 11$ $\quad d\nu = 0$, $\quad d\widetilde{\nu} = 0$.

2. Curvatures on $\mathcal{W}_0$'s hypersurfaces of second type

Let (M', J, g'), $\dim M' = 2n' = 2n + 2$, be an almost complex manifold with B-metric. We determine a $(2n + 1)$-dimensional differentiable hypersurface M embedded in M' by the condition $M : \tilde{g}'(Z, Z) = 0$ for a vector field Z on M'. It is clear that $g'(Z, JZ) = 0$. At every point we put $g'(Z, Z) = \cosh^2 t$, $t > 0$ for the sake of the impossibility Z to be a main isotropic direction and in view of definiteness.

We choose the time-like unit normal $N = \frac{1}{\cosh t}JZ$, i.e. $g'(N, N) = -1$. Hence, JN is a space-like unit tangent vector field on M.

Let us recall the following

Definition 1. [[?]] The hypersurface M of an almost complex manifold with B-metric (M', J, g'), determined by the condition the normal unit N

to be isotropic regarding the associated B-metric $\widetilde{g'}$ of g', equipped with the almost contact B-metric structure

$$\varphi := J + g'(\cdot, JN)N, \quad \xi := -JN, \quad \eta := -g'(\cdot, JN), \quad g := g'|_M$$

will be called *a hypersurface of second type* of (M', J, g').

In the case when (M', J, g') is a Kaehlerian manifold with B-metric (i.e. a $\mathcal{W}_0$-manifold), in [**?**] it is ascertained that every hypersurface of second type belongs to the class $\mathcal{F}_4 \oplus \mathcal{F}_5 \oplus \mathcal{F}_6 \oplus \mathcal{F}_8$ and

$$\begin{gathered} F(X,Y,Z) = g(AX,Y)\eta(Z) + g(AX,Z)\eta(Y), \\ \theta(Z) = \operatorname{tr} A.\eta(Z), \quad \theta^*(Z) = \operatorname{tr}(A\circ\varphi)\eta(Z), \quad \omega(Z) = 0. \end{gathered} \tag{2.1}$$

The second fundamental form of the hypersurfaces in consideration is $AX = -\varphi\nabla_X\xi$. The mentioned classes of hypersurfaces of second type are characterized by A as follows [**?**]:

$$\begin{array}{ll} \mathcal{F}_0: & A = 0; \quad \mathcal{F}_4: \quad A = -\frac{\theta(\xi)}{2n}\varphi^2; \quad \mathcal{F}_5: \quad A = -\frac{\theta^*(\xi)}{2n}\varphi; \\ \mathcal{F}_6: & A\circ\varphi = \varphi\circ A\,, \quad A\xi = 0\,, \quad \operatorname{tr}A = \operatorname{tr}(A\circ\varphi) = 0; \\ \mathcal{F}_8: & A\circ\varphi = -\varphi\circ A\,, \quad A\xi = 0. \end{array}$$

For the curvature tensors R' and R of the Kaehlerian manifold (M', J, g') and of its hypersurface of second type $(M, \varphi, \xi, \eta, g)$, respectively, we have

$$R'(x,y,z,u) = R(x,y,z,u) + \pi_1(Ax,Ay,z,u), \;\; R'(x,y)N = -\left(\nabla_x A\right)y + \left(\nabla_y A\right)x.$$

Hence and because of Theorem **??** we obtain:

$$\begin{aligned} R(x,y,z,u) &= \left\{\nu'\left[\pi_1' - \pi_2'\right] + \widetilde{\nu'}\pi_3'\right\}(x,y,z,u) - \pi_1(Ax,Ay,z,u), \\ R(x,y,\varphi z,\varphi u) &= -\left\{R - \nu'\pi_4 - \widetilde{\nu'}\pi_5\right\}(x,y,z,u) - \left[\pi_1 + \pi_2\right](Ax,Ay,z,u), \\ R(x,y)\xi &= \left[\nu'\pi_4 - \widetilde{\nu'}\pi_5\right](x,y)\xi, \quad R'(x,y)N = \left[\nu'\pi_5 + \widetilde{\nu'}\pi_4\right](x,y)\xi. \end{aligned}$$

Therefore

$$\left(\nabla_x A\right)y - \left(\nabla_y A\right)x = -\nu'\pi_5(x,y)\xi - \widetilde{\nu'}\pi_4(x,y)\xi. \tag{2.2}$$

Having in mind that for hypersurfaces of second type are valid the equations: $g'(y,Jz) = g(y,\varphi z)$, $\pi_1' = \pi_1$, $\pi_2' = \pi_2$, $\pi_3' = \pi_3$, we obtain the following

Proposition 1. A hypersurface of second type of a Kaehlerian manifold with B-metric has the following curvature properties:

$$R(x,y,z,u) = \left\{\nu'\left[\pi_1-\pi_2\right]+\widetilde{\nu'}\pi_3\right\}(x,y,z,u)-\pi_1(Ax,Ay,z,u),$$
$$\tau = 4n^2\nu' - (\operatorname{tr} A)^2 + \operatorname{tr} A^2, \quad \tau^* = 2n\,(2n-1)\,\widetilde{\nu'} - \operatorname{tr} A.\operatorname{tr}(A\circ\varphi) + \operatorname{tr}\left(A^2\circ\varphi\right),$$

for a ξ-section $\{\xi, x\}$ the sectional curvature k doesn't depend on A

$$k\,(\xi,x) = \nu' - \widetilde{\nu'}\frac{g(x,\varphi x)}{g(x,x)-[\eta(x)]^2},$$

for a φ-holomorphic section $\left\{\varphi x, \varphi^2 x\right\}$ and for a totally real section $\{x, y\}$, orthogonal to ξ, respectively:

$$k\left(\varphi x,\varphi^2 x\right) = \frac{\pi_1(Ax,A\varphi x,\varphi x,x)}{\pi_1(\varphi x,\varphi^2 x,\varphi^2 x,\varphi x)}, \quad k\,(x,y) = \nu' - \frac{\pi_1(Ax,Ay,y,x)}{\pi_1(x,y,y,x)}.$$

When (M,φ,ξ,η,g) is a hypersurface of second type of $\mathcal{W}_0$-manifold, the equations (**??**) and (**??**) imply

$$K(x,y,z,u) = R(x,y,\varphi^2 z,\varphi^2 u) + \pi_1(Ax,Ay,\varphi z,\varphi u).$$

If (M',J,g') has constant totally real sectional curvatures in addition, then for K we obtain the following

Proposition 2. Let (M,φ,ξ,η,g) be a hypersurface of second type of a $\mathcal{W}_0$-manifold (M',J,g') with constant totally real sectional curvatures. Then K of (M,φ,ξ,η,g) is Kaehlerian and

$$K(x,y,z,u) = \left\{\nu'\left[\pi_1-\pi_2-\pi_4\right]+\widetilde{\nu'}\left[\pi_3+\pi_5\right]\right\}(x,y,z,u) - \left[\pi_1-\pi_2\right](Ax,Ay,z,u),$$
$$\tau(K) = 4n(n-1)\nu' - \left[\theta^2(\xi)-\theta^{*2}(\xi)\right] - \operatorname{tr}\left(\varphi\circ A\circ\varphi\circ A\right) + \operatorname{tr} A^2,$$
$$\widetilde{\tau}(K) = -4n(n-1)\widetilde{\nu'} + 2\theta(\xi)\theta^*(\xi) - \operatorname{tr}\left(\varphi\circ A^2\right) - \operatorname{tr}\left(A\circ\varphi\circ A\right).$$

3. Curvatures on $\mathcal{W}_0$'s hypersurfaces of second type, belonging to the main classes

Now, let (M,φ,ξ,η,g) belong to the widest main class $\mathcal{F}_4\oplus\mathcal{F}_5$ of hypersurfaces of second type. Let us recall that a class of almost contact B-metric manifolds is said to be main if the tensor F is expressed explicitly by the structural tensors φ,ξ,η,g. So, in this class for the second fundamental tensor we have

$$A=-\frac{1}{2n}\left[\theta(\xi)\varphi^2+\theta^*(\xi)\varphi\right],\quad \operatorname{tr}A=\theta(\xi),\quad \operatorname{tr}(A\circ\varphi)=\theta^*(\xi). \tag{3.1}$$

Having in mind Proposition **??**, we obtain the next

Corollary 1. If a hypersurface of second type of a Kaehlerian manifold with B-metric is $\mathcal{F}_4\oplus\mathcal{F}_5$-manifold, then it has the following curvature properties:

$$\begin{aligned}
R&=\nu'\left[\pi_1-\pi_2\right]+\widetilde{\nu'}\pi_3\\
&\quad-\frac{1}{4n^2}\left\{\theta^2(\xi)\left[\pi_1-\pi_4\right]+\theta^{*2}(\xi)\pi_2+\theta(\xi)\theta^*(\xi)\left[\pi_3+\pi_5\right]\right\},\\
\tau&=4n^2\nu'-\frac{1}{2n}\left[(2n-1)\,\theta^2(\xi)+\theta^{*2}(\xi)\right],\\
\tau^*&=2n\,(2n-1)\,\widetilde{\nu'}-\frac{n-1}{n}\theta(\xi)\theta^*(\xi),
\end{aligned}$$

$$k\left(\varphi x,\varphi^2x\right)=-\frac{\theta^2(\xi)+\theta^{*2}(\xi)}{4n^2},\quad k\,(x,y)=\nu'-\frac{\theta^2(\xi)}{4n^2},\qquad \text{where } g^*(\cdot,\cdot)=g(\cdot,\varphi\cdot).$$

Remark 1. We can obtain the corresponding properties for the classes $\mathcal{F}_4$, $\mathcal{F}_5$ and $\mathcal{F}_0$, if we substitute $\theta^*(\xi)=0$, $\theta(\xi)=0$ and $\theta(\xi)=\theta^*(\xi)=0$, respectively.

Using the equations (**??**) and (**??**), we express the canonical connection explicitly for the class $\mathcal{F}_4\oplus\mathcal{F}_5$

$$D_XY=\nabla_XY+\frac{\theta(\xi)}{2n}\left\{g(x,\varphi y)\xi-\eta(y)\varphi x\right\}-\frac{\theta^*(\xi)}{2n}\left\{g(\varphi x,\varphi y)\xi-\eta(y)\varphi^2x\right\}$$

It is clear that if the hypersurface in consideration belongs to $\mathcal{F}_4\oplus\mathcal{F}_5$, then it is $\left(\mathcal{F}_4^0\oplus\mathcal{F}_5^0\right)$-manifold. Let $(M,\varphi,\xi,\eta,g)\in\mathcal{F}_4^0\oplus\mathcal{F}_5^0$, i.e. (M,φ,ξ,η,g)

is $(\mathcal{F}_4 \oplus \mathcal{F}_5)$-manifold with closed 1-forms θ and θ^*. The canonical curvature tensor K of every $\left(\mathcal{F}_4^0 \oplus \mathcal{F}_5^0\right)$-manifold is Kaehlerian and it has the appearance

$$\begin{aligned} K = R &+ \frac{\xi\theta(\xi)}{2n}\pi_5 + \frac{\xi\theta^*(\xi)}{2n}\pi_4 \\ &+ \frac{\theta^2(\xi)}{4n^2}\left[\pi_2 - \pi_4\right] + \frac{\theta^{*2}(\xi)}{4n^2}\pi_1 - \frac{\theta(\xi)\theta^*(\xi)}{4n^2}\left[\pi_3 - \pi_5\right]. \end{aligned}$$

Then, using Corollary **??**, we ascertain the truthfulness of the following

Corollary 2. If the manifold is a hypersurface of second type of a Kaehlerian manifold with B-metric and constant totally real sectional curvatures, then K is expressed by the following way:

$$\begin{aligned} K = &\left[\nu' - \frac{\theta^2(\xi) - \theta^{*2}(\xi)}{4n^2} + \frac{\xi\theta^*(\xi)}{2n}\right](\pi_1 - \pi_2) + \left[\widetilde{\nu'} - \frac{2\theta(\xi)\theta^*(\xi)}{4n^2} - \frac{\xi\theta(\xi)}{2n}\right]\pi_3 \\ &- \frac{\xi\theta^*(\xi)}{2n}(\pi_1 - \pi_2 - \pi_4) + \frac{\xi\theta(\xi)}{2n}(\pi_3 + \pi_5). \end{aligned}$$

We compute the expression $(\nabla_x A)\, y - (\nabla_y A)\, x$ using (**??**) and we compare the result with (**??**). Thus we get the relations

$$\nu' = -\frac{\xi\theta^*(\xi)}{2n} + \frac{\theta^2(\xi) - \theta^{*2}(\xi)}{4n^2}, \quad \widetilde{\nu'} = \frac{\xi\theta(\xi)}{2n} + \frac{2\theta(\xi)\theta^*(\xi)}{4n^2}. \tag{3.2}$$

Therefore K is Kaehlerian and

$$\begin{aligned} K &= -\frac{\xi\theta^*(\xi)}{2n}\left[\pi_1 - \pi_2 - \pi_4\right] + \frac{\xi\theta(\xi)}{2n}\left[\pi_3 + \pi_5\right], \\ R &= -\frac{\xi\theta^*(\xi)}{2n}\left[\pi_1 - \pi_2\right] + \frac{\xi\theta(\xi)}{2n}\pi_3 \\ &\quad - \frac{\theta^2(\xi)}{4n^2}\left[\pi_2 - \pi_4\right] - \frac{\theta^{*2}(\xi)}{4n^2}\pi_1 + \frac{\theta(\xi)\theta^*(\xi)}{4n^2}\left[\pi_3 - \pi_5\right]. \end{aligned}$$

We solve the system (**??**) with respect to the functions $\theta(\xi)$ and $\theta^*(\xi)$ and we get

$$\theta(\xi) = \pm 2n\sqrt{\frac{a}{2}}, \quad \theta^*(\xi) = \pm 2n\frac{\widetilde{\nu'}}{\sqrt{2a}}, \quad a = \nu' + \sqrt{\nu'^2 + \widetilde{\nu'}^2}. \tag{3.3}$$

Since ν' and $\widetilde{\nu'}$ are pointly constant for M'^4 $(n = 1)$ and they are absolute constants for M'^{2n+2} $(n \geq 2)$ (Theorem **??**), then the functions $\theta(\xi)$ and

$\theta^*(\xi)$, which determine the hypersurface of second type as an almost contact B-metric manifold, are also pointly constant on M^3 and constants on M^5.

Hence for $n \geq 2$ we have

Theorem 4. Every $\left(\mathcal{F}_4^0 \oplus \mathcal{F}_5^0\right)$-manifold of dimension at least 5, as a hypersurface of second type with 1-forms $\theta = \theta(\xi)\,\eta$ and $\theta^* = \theta^*(\xi)\,\eta$ of a Kaehlerian manifold with B-metric and constant totally real sectional curvatures $\nu'$and $\widetilde{\nu'}$ has $K = 0$ and the following curvature properties for R:

$$\begin{aligned}
R &= -\frac{\theta^2(\xi)}{4n^2}\left[\pi_2 - \pi_4\right] - \frac{\theta^{*2}(\xi)}{4n^2}\pi_1 + \frac{\theta(\xi)\theta^*(\xi)}{4n^2}\left[\pi_3 - \pi_5\right] \\
&= -\frac{a}{2}\left[\pi_2 - \pi_4\right] - \frac{\widetilde{\nu'}^2}{2a}\pi_1 + \frac{\widetilde{\nu'}}{2}\left[\pi_3 - \pi_5\right], \\
\tau(R) &= \frac{\theta^2(\xi)}{2n} - (2n+1)\,\frac{\theta^{*2}(\xi)}{2n} = n.a - (2n+1)\,\frac{n.\widetilde{\nu'}^2}{a},
\end{aligned}$$

$$\begin{aligned}
\tau^*(R) &= \theta(\xi)\theta^*(\xi) = 2n^2\widetilde{\nu'}, \\
k\left(\xi, x\right) &= \frac{\theta^2(\xi) - \theta^{*2}(\xi)}{4n^2} + \frac{2\theta(\xi)\theta^*(\xi)}{4n^2}\frac{g(x,\varphi x)}{g(\varphi x,\varphi x)} = \nu' + \widetilde{\nu'}\frac{g(x,\varphi x)}{g(\varphi x,\varphi x)} \\
k\left(\varphi x, \varphi^2 x\right) &= -\frac{\theta^2(\xi) + \theta^{*2}(\xi)}{4n^2} = -\nu' - \frac{\widetilde{\nu'}^2}{a} = const, \\
k\left(x, y\right) &= -\frac{\theta^{*2}(\xi)}{4n^2} = -\frac{\widetilde{\nu'}^2}{2a} = const, \qquad \text{where } a = \nu' + \sqrt{\nu'^2 + \widetilde{\nu'}^2}.
\end{aligned}$$

References

[1] A. Borisov and G. Ganchev, *Curvature properties of Kaehlerian manifolds with B-metric*, Math. and Educ. in Math., Proc. of 14-th Spring Conference of UBM, Sunny Beach, 1985, 220–226.

[2] G. Ganchev and A. Borisov, *Note on the almost complex manifolds with Norden metric*, C. R. Acad. Bulgare Sci., **39** (1986), no. 5, 31–34.

[3] G. Ganchev, K. Gribachev and V. Mihova, *Holomorphic hypersurfaces of Kaehlerian manifolds with Norden metric*, Plovdiv. Univ. Paisiĭ Khilendarski Nauchn. Trud. Mat., **23** (1985), no. 2, 221–236.

[4] G. Ganchev, V. Mihova and K. Gribachev, *Almost contact manifolds with B-metric*, Math. Balkanica (N.S.), **7** (1993), no. 3-4, 261–276.

[5] M. Manev, *Properties of curvatures tensors on almost contact manifolds with B-metric*, Proc. of Jubilee Sci. Session of V. Levsky Higher Military school, V. Tarnovo, **27** (1993), 221–227.

[6] M. Manev, *Almost contact B-metric hypersurfaces of Kaehlerian manifolds with B-metric*, (to appear).

[7] M. Manev and K. Gribachev, *Conformally invariant tensors on almost contact manifolds with B-metric*, Serdica - Bulg. Math. Publ., **20** (1994), 133–147.

[8] G. Nakova and K. Gribachev, *Submanifolds of some almost contact manifolds with B-metric with codimension two. I*, Math. Balkanica (N.S.), **12** (1998), no. 1-2, 93–108.

[9] A. P. Norden, On a class of four-dimensional A-spaces (in Russian), Izv. Vyssh. Uchebn. Zaved. Mat., **17** (1960), no. 4, 145–157.

Faculty of Mathematics and Informatics
University of Plovdiv
236 Bulgaria blvd.
Plovdiv 4004, Bulgaria

Received 19 September 2000