Curvature Tensors on Hypersurfaces of a Kähler Manifold with B-Metric

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Abstract

The curvature tensor in the basic classes of the real time-like hypersurfaces of a Kähler manifold with B-metric is given. The essential components of these curvature tensors with respect to the standard complex coordinates are obtained. The case of a Kähler manifold with B-metric and constant totally real sectional curvatures is considered. These hypersurfaces are characterized in terms of the curvature tensor.

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1 Preliminaries

In [5] two types of real hypersurfaces of a complex manifold with B-metric are introduced. The obtained submanifolds are almost contact B-metric manifolds. The real time-like hypersurface of an almost complex manifold with B-metric is determined by the condition the normal unit to be time-like. The class of such hypersurfaces of a Kähler manifold with B-metric as almost contact B-metric manifolds is determined in [7].

In this paper we study the curvature tensor of the real time-like hypersurfaces of a Kähler manifold with B-metric and constant totally real sectional curvatures. We give the essential components and the complex equalities for the curvature tensor and we characterize these hypersurfaces by their curvature tensor according to the decomposition from [9].

Let (M', J, g') be a $(2n+2)$-dimensional almost complex manifold with B-metric, i.e. J is an almost complex structure and g' is a metric on M' such that: $J^2 = -Id$, $g'(J\cdot, J\cdot) = -g'(\cdot,\cdot)$. The associated metric $\tilde{g}'$ of g' is given by $\tilde{g}'(\cdot,\cdot) = g'(\cdot, J\cdot)$. Both metrics are necessarily of signature $(n+1, n+1)$ [2].

The class $\mathcal{W}_0$ of the Kähler manifolds with B-metric is determined by the condition $\nabla' J = 0$, where ∇' is the Levi-Civita connection of g'.

Let $(M, \varphi, \xi, \eta, g)$ be a $(2n+1)$-dimensional almost contact manifold with B-metric, i.e. (φ, ξ, η) is an almost contact structure and g is a metric on M such that: $\varphi^2 = -id + \eta \otimes \xi$; $\eta(\xi) = 1$; $g(\varphi\cdot, \varphi\cdot) = -g(\cdot,\cdot) + \eta(\cdot)\eta(\cdot)$. Both metrics g and its associated $\tilde{g} : \tilde{g}(\cdot,\cdot) = g(\cdot, \varphi\cdot) + \eta(\cdot)\eta(\cdot)$ are indefinite metrics of signature $(n+1, n)$ [3].

Further, X, Y, Z will stand for arbitrary differentiable vector fields on M and x, y, z – arbitrary vectors in the tangential space T_pM to M at some point p in M.

A decomposition of the class of the almost contact manifolds with B-metric with respect to the tensor $F: F(X,Y,Z) = g((\nabla_X\varphi)Y, Z)$ is given in [3], where are defined eleven basic classes $\mathcal{F}_i$ $(i = 1, \dots, 11)$. The Levi-Civita connection of g is denoted by ∇. The special class $\mathcal{F}_0$: $F = 0$ is contained in each of $\mathcal{F}_i$. The following 1-forms are associated with F: $\theta(\cdot) = g^{ij}F(e_i, e_j, \cdot)$, $\theta^*(\cdot) = g^{ij}F(e_i, \varphi e_j, \cdot)$, $\omega(\cdot) = F(\xi, \xi, \cdot)$, where $\{e_i, \xi\}$ $(i = 1, \dots, 2n)$ is a basis of T_pM, and (g^{ij}) is the inverse matrix of (g_{ij}).

Let $M : g'(N,N) = -1$ be a real time-like hypersurface of (M', J, g') and let M be equipped with the almost contact B-metric structure as in [5]

$$\begin{aligned} &\varphi := J + \cos t.g'(\cdot, JN)\{\cos t.N - \sin t.JN\}, \quad \xi := \sin t.N + \cos t.JN, \\ &\eta := \cos t.g'(\cdot, JN), \quad g := g'|_M, \quad t := \arctan\{g'(N, JN)\}, \quad t \in \left(-\tfrac{\pi}{2}; \tfrac{\pi}{2}\right). \end{aligned}$$

In [7] it is determined the class of real time-like hypersurfaces of a $\mathcal{W}_0$-manifold in terms of F as $\mathcal{F}_4 \oplus \mathcal{F}_5 \oplus \mathcal{F}_6 \oplus \mathcal{F}_{11}$.

If $h(X,Y) = g(AX, Y)$ is the second fundamental form of the hypersurface M, then the formulas of Gauss and Weingarten in this case are:

$$\nabla'_X Y = \nabla_X Y - h(X,Y)N, \qquad \nabla'_X N = -AX. \tag{1}$$

In [9] it is obtained an orthogonal and invariant decomposition of the space of the $(0,2)$-tensors over a vector space with (φ, ξ, η, g)-structure to nine subspaces $\mathcal{L}_1, \dots, \mathcal{L}_9$. Using this decomposition and the results for h from [5] and [7], we obtain the following

Theorem 1.1 *The basic classes of real time-like hypersurfaces of a $\mathcal{W}_0$-manifold are characterized in terms of their second fundamental form h as follows:*

$$\mathcal{F}_0: \; h = h_0; \qquad \mathcal{F}_i: \; h = h_0 + h_i \qquad (i = 4, 5, 6, 11),$$

where $h_0 \in \mathcal{L}_9$; $h_4, h_5 \in \mathcal{L}_1 \oplus \mathcal{L}_2$; $h_6 \in \mathcal{L}_3$; $h_{11} \in \mathcal{L}_7$ and

$$\begin{aligned} &h_0(X,Y) = -\tfrac{dt(\xi)}{2\cos t}\eta(X)\eta(Y), \qquad h_4(X,Y) = \tfrac{\theta(\xi)}{2n}\{\cos t.g(\varphi X, \varphi Y) - \sin t.g(X, \varphi Y)\}, \\ &h_5(X,Y) = \tfrac{\theta^*(\xi)}{2n}\{\sin t.g(\varphi X, \varphi Y) + \cos t.g(X, \varphi Y)\}, \\ &h_6(X,Y) = -\tfrac{1}{2}[\sin t.(\mathcal{L}_\xi g)(X,Y) - \cos t.(\mathcal{L}_\xi g)(X, \varphi Y)], \\ &\qquad\qquad (\mathcal{L}_\xi g)(X,Y) = (\nabla_X \eta)Y + (\nabla_Y \eta)X, \\ &h_{11}(X,Y) = -\sin t[\eta(X)\omega(\varphi Y) + \eta(Y)\omega(\varphi X)] - \cos t[\eta(X)\omega(Y) + \eta(Y)\omega(X)]. \end{aligned}$$

2 Curvature tensors

In this section we express the curvature tensor in the basic classes of the real time-like hypersurfaces of a Kähler manifold with B-metric.

The curvature tensor R for ∇ is defined as ordinary by $R = [\nabla, \nabla] - \nabla_{[\ ,\ ]}$. The tensor R' is corresponding curvature tensor for ∇'.

It is known [1], that a $\mathcal{W}_0$-manifold ($\dim M' \geq 4$) is of constant sectional curvatures ν' and $\tilde{\nu}'$ for nondegenerate totally real sections α' (i.e. $J\alpha' \perp \alpha'$) if and only if

$$R' = \nu' \left[\pi_1' - \pi_2'\right] + \tilde{\nu}'\pi_3'. \tag{2}$$

Both functions ν' and $\tilde{\nu}'$ are constant if M' is connected and $\dim M' \geq 6$.

We use the following curvature-like tensors of type $(0,4)$, which are invariant with respect to the structural group. The tensor S is a symmetric and φ-antiinvariant tensor of type $(0,2)$.

$$\begin{aligned}
\psi_1(S)(x,y,z,u) &= g(y,z)S(x,u) - g(x,z)S(y,u) + g(x,u)S(y,z) - g(y,u)S(x,z),\\
\psi_2(S)(x,y,z,u) &= \psi_1(S)(x,y,\varphi z,\varphi u),\\
\psi_3(S)(x,y,z,u) &= -\psi_1(S)(x,y,\varphi z,u) - \psi_1(S)(x,y,z,\varphi u),\\
\psi_4(S)(x,y,z,u) &= \psi_1(S)(x,y,\xi,u)\eta(z) + \psi_1(S)(x,y,z,\xi)\eta(u),\\
\psi_5(S)(x,y,z,u) &= \psi_1(S)(x,y,\xi,\varphi u)\eta(z) + \psi_1(S)(x,y,\varphi z,\xi)\eta(u).
\end{aligned}$$

We denote tensors $\pi_i = \frac{1}{2}\psi_i(g)$ $(i=1,2,3), \pi_i = \psi_i(g)$ $(i=4,5)$. The tensor $\pi_i(S)$ is π_i with the substitution S for g, and π_i' is π_i with respect to g' ($i = 1,2,3$, respectively).

Having in mind the equations (1), (2), Theorem 1.1, the Gauss equation and the Codazzi-Mainardi equation, we obtain the following

Theorem 2.1 *Let M be a real time-like hypersurface of a $\mathcal{W}_0$-manifold with constant totally real sectional curvatures ν' and $\tilde{\nu}'$. Then*

$$\begin{aligned}
R(x,y,z,u) = &\left\{\nu'\left[\pi_1 - \pi_2 - \tan t.\pi_5\right] + \tilde{\nu}'\left[\pi_3 - \tan t.\pi_4\right]\right\}(x,y,z,u)\\
&-\pi_1(Ax,Ay,z,u),\\
(\nabla_x h)(y,z) - (\nabla_y h)(x,z) = &-\tfrac{1}{\cos t}\left\{\nu'\pi_5 + \tilde{\nu}'\pi_4\right\}(x,y)z
\end{aligned}$$

and the tensor $\pi_1(Ax,Ay,z,u)$ takes the following form in the basic classes:

	$\pi_1(Ax,Ay,z,u) =$
$\mathcal{F}_0$	$= 0$
$\mathcal{F}_4$	$= \frac{\theta(\xi)dt(\xi)}{4n\cos t}\left\{\sin t.\pi_5 + \cos t.\pi_4\right\}$ $+\frac{\theta^2(\xi)}{4n^2}\left\{\cos^2 t\left[\pi_1 - \pi_4\right] + \sin^2 t.\pi_2 - \sin t\cos t\left[\pi_3 + \pi_5\right]\right\}$
$\mathcal{F}_5$	$= -\frac{\theta^*(\xi)dt(\xi)}{4n\cos t}\left\{\cos t.\pi_5 - \sin t.\pi_4\right\}$ $+\frac{\theta^{*2}(\xi)}{4n^2}\left\{\sin^2 t\left[\pi_1 - \pi_4\right] + \cos^2 t.\pi_2 + \sin t\cos t\left[\pi_3 + \pi_5\right]\right\}$
$\mathcal{F}_6$	$= \frac{dt(\xi)}{2\cos t}\left[\sin t.\psi_4\left(\mathcal{L}_\xi g\right) - \cos t.\psi_5\left(\mathcal{L}_\xi g\right)\right]$ $+\frac{1}{4}\left[\sin^2 t.\pi_1\left(\mathcal{L}_\xi g\right) + \cos^2 t.\pi_2\left(\mathcal{L}_\xi g\right) + \sin t\cos t.\pi_3\left(\mathcal{L}_\xi g\right)\right]$
$\mathcal{F}_{11}$	$= -\cos^2 t.\psi_4(\omega\otimes\omega) - \sin^2 t.\psi_4(\tilde{\omega}\otimes\tilde{\omega})$ $-\sin t\cos t.\psi_4(\omega\otimes\tilde{\omega} + \tilde{\omega}\otimes\omega), \qquad \tilde{\omega} = \omega\circ\varphi$

Having in mind the Kähler property of R in the class $\mathcal{F}_0$ [4], we obtain the following

Corollary 2.2 *Let M be a time-like hypersurface of a $\mathcal{W}_0$-manifold M' with constant totally real sectional curvatures. If M belongs to the class $\mathcal{F}_0$ then M is a hyperplane of the flat manifold M'.*

3 The essential components of the curvature tensor

In this section we get the essential (which may be nonzero) components of the curvature tensors in the basic classes of the real time-like hypersurfaces of a Kähler manifold with B-metric.

Let $(V^{2n+1}, \varphi, \xi, \eta, g)$ be an almost contact vector space with B-metric and $V^{\mathbb{C}} = D^{\mathbb{C}} \oplus \{\xi\}$, where $D^{\mathbb{C}}$ is the complexification of $D = \ker \eta$ and $\{\xi\} = (\operatorname{im} \eta)\xi$. It is valid the decomposition $D^{\mathbb{C}} = D^{1,0} \oplus D^{0,1}$, where $D^{1,0}$ and $D^{0,1}$ are the i-eigenspace and the $(-i)$-eigenspace of φ, respectively. Moreover, we have $D^{1,0} = \operatorname{span}\{Z_\alpha = e_\alpha - i\varphi e_\alpha\}$, $D^{0,1} = \operatorname{span}\{Z_{\bar\alpha} = e_\alpha + i\varphi e_\alpha\}$, $Z_0 \equiv \xi$ for an orthonormal basis $\{e_\alpha, \varphi e_\alpha, \xi\}_{\alpha=1}^n$ of V.

Let $(M, \varphi, \xi, \eta, g)$ be a real time-like hypersurfaces of a $\mathcal{W}_0$-manifold. Following [8] we give the following

Lemma 3.1 *The characteristic conditions for the basic classes of the considered hypersurfaces in terms of the essential components and the essential complex equalities for the fundamental tensors are the following:*

$$\begin{aligned}
&\mathcal{F}_4: \ h_{\alpha\beta} = -e^{it}F_{\alpha\beta 0} = -e^{it}F_{\beta\alpha 0} = -e^{it}\tfrac{\theta(\xi)}{2n}g_{\alpha\beta}, \quad h_{00} = -\tfrac{dt(\xi)}{2\cos t};\\
&\mathcal{F}_5: \ h_{\alpha\beta} = -e^{it}F_{\alpha\beta 0} = -e^{it}F_{\beta\alpha 0} = ie^{it}\tfrac{\theta^*(\xi)}{2n}g_{\alpha\beta}, \quad h_{00} = -\tfrac{dt(\xi)}{2\cos t};\\
&\mathcal{F}_6: \ h_{\alpha\beta} = -e^{it}F_{\alpha\beta 0} = -e^{it}F_{\beta\alpha 0} = \tfrac{i}{2}e^{it}(\mathcal{L}_\xi g)_{\alpha\beta}, \quad \operatorname{tr}h = h_{00} = -\tfrac{dt(\xi)}{2\cos t}, \quad tr^*h = 0;\\
&\mathcal{F}_{11}: \ h_{0\alpha} = -e^{it}\omega_\alpha = -ie^{it}(\mathcal{L}\eta)_{\alpha 0} = -\tfrac{i}{2}e^{it}(\mathcal{L}g)_{\alpha 00}, \quad h_{00} = -\tfrac{dt(\xi)}{2\cos t}.
\end{aligned}$$

The Codazzi-Mainardi equation implies the following

Lemma 3.2 *The essential components of the covariant derivative of the second fundamental form h of M satisfy the following equalities:*

$$\begin{aligned}
&(\nabla_\alpha h)_{bc} - (\nabla_b h)_{\alpha c} = 0, \quad b \in \{\beta, \bar\beta\}, \quad c \in \{\gamma, \bar\gamma, 0\};\\
&(\nabla_0 h)_{\alpha b} - (\nabla_\alpha h)_{0b} = 0, \quad b \in \{\bar\beta, 0\};\\
&(\nabla_0 h)_{\alpha\beta} - (\nabla_\alpha h)_{0\beta} = -\tfrac{1}{\cos t}(\tilde\nu' + i\nu')g_{\alpha\beta}.
\end{aligned}$$

Having in mind the Gauss equation we get

Lemma 3.3 *The essential components of the curvature tensor R of M are:*

$$\begin{aligned}
&R_{\alpha\beta\gamma\delta} = 2\nu'(g_{\beta\gamma}g_{\alpha\delta} - g_{\alpha\gamma}g_{\beta\delta}) + 2i\tilde\nu'(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) - h_{\beta\gamma}h_{\alpha\delta} + h_{\alpha\gamma}h_{\beta\delta},\\
&R_{\alpha\beta\bar\gamma\bar\delta} = -h_{\beta\bar\gamma}h_{\alpha\bar\delta} + h_{\alpha\bar\gamma}h_{\beta\bar\delta}, \qquad R_{\bar\alpha\beta\gamma\delta} = -h_{\beta\gamma}h_{\bar\alpha\delta} + h_{\bar\alpha\gamma}h_{\beta\delta},\\
&R_{\alpha\bar\beta\gamma\bar\delta} = -h_{\bar\beta\gamma}h_{\alpha\bar\delta} + h_{\alpha\gamma}h_{\bar\beta\bar\delta}, \qquad R_{\alpha\beta\gamma 0} = -h_{\beta\gamma}h_{\alpha 0} + h_{\alpha\gamma}h_{\beta 0},\\
&R_{\bar\alpha\beta\gamma 0} = -h_{\beta\gamma}h_{\bar\alpha 0} + h_{\bar\alpha\gamma}h_{\beta 0}, \qquad R_{\alpha\beta\bar\gamma 0} = -h_{\beta\bar\gamma}h_{\alpha 0} + h_{\alpha\bar\gamma}h_{\beta 0},\\
&R_{0\alpha\beta 0} = (\nu' - \tilde\nu'\tan t)g_{\alpha\beta} - i(\nu'\tan t + \tilde\nu')g_{\alpha\beta} - h_{\alpha\beta}h_{00} + h_{0\beta}h_{\alpha 0},\\
&R_{0\bar\alpha\beta 0} = -h_{\bar\alpha\beta}h_{00} + h_{0\beta}h_{\bar\alpha 0}.
\end{aligned}$$

A decomposition of the set $\mathcal{R}$ of all curvature-like tensors over $(V, \varphi, \xi, \eta, g)$ into 20 mutually orthogonal and invariant factors with respect to the structural group $\big(GL(n, \mathbb{C}) \cap O(n, n)\big) \times I$ is obtained in [9]. Each of these factors is described by the essential components and the complex equalities for R with respect to the standard complex coordinates in [10]. Let us recall, an almost contact manifold with B-metric is said to be in the class $h\mathcal{R}_i$, $h\mathcal{R}_i^{\perp}$, $v\mathcal{R}_j$, $v\mathcal{R}_j^{\perp}$, $w\mathcal{R}$, ω_k, v_r, w_s if $R \in h\mathcal{R}_i$, $h\mathcal{R}_i^{\perp}$, $v\mathcal{R}_j$, $v\mathcal{R}_j^{\perp}$, $w\mathcal{R}$, ω_k, v_r, w_s, respectively, where $i = 1,2,3$; $j = 1,2$; $k = 1,\dots,11$; $r = 1,\dots,5$; $s = 1,\dots,4$. Using the characterization from [10] of the twenty factors of $\mathcal{R}$, Lemma 3.1 and Lemma 3.3, we express the essential components and the decomposition of R in the following

Theorem 3.4 *Let M be a real time-like hypersurface of a $\mathcal{W}_0$-manifold of constant totally real sectional curvatures ν' and $\tilde{\nu}'$. Then for the basic classes of the considered hypersurfaces we have:*

$\mathcal{F}_4$: $M \in \omega_1 \oplus \omega_2 \oplus \omega_5 \oplus w_1 \oplus w_2$,

$$R_{\alpha\beta\gamma\delta} = 2\left\{\left[\nu' - \frac{\theta^2(\xi)}{8n^2}\cos 2t\right] - i\left[\tilde{\nu}' + \frac{\theta^2(\xi)}{8n^2}\sin 2t\right]\right\}(g_{\beta\gamma}g_{\alpha\delta} - g_{\alpha\gamma}g_{\beta\delta}),$$
$$R_{\alpha\bar{\beta}\gamma\bar{\delta}} = 2\frac{\theta^2(\xi)}{8n^2}g_{\alpha\gamma}g_{\bar{\beta}\bar{\delta}},$$
$$R_{0\alpha\beta0} = \left[\nu' - \tilde{\nu}'\tan t - \frac{\theta(\xi)}{4n}dt(\xi)\right]g_{\alpha\beta} - i\left[\nu'\tan t + \tilde{\nu}' + \frac{\theta(\xi)}{4n}dt(\xi)\tan t\right]g_{\alpha\beta};$$

$\mathcal{F}_5$: $M \in \omega_1 \oplus \omega_2 \oplus \omega_5 \oplus w_1 \oplus w_2$,

$$R_{\alpha\beta\gamma\delta} = 2\left\{\left[\nu' + \frac{\theta^{*2}(\xi)}{8n^2}\cos 2t\right] - i\left[\tilde{\nu}' - \frac{\theta^{*2}(\xi)}{8n^2}\sin 2t\right]\right\}(g_{\beta\gamma}g_{\alpha\delta} - g_{\alpha\gamma}g_{\beta\delta}),$$
$$R_{\alpha\bar{\beta}\gamma\bar{\delta}} = 2\frac{\theta^{*2}(\xi)}{8n^2}g_{\alpha\gamma}g_{\bar{\beta}\bar{\delta}},$$
$$R_{0\alpha\beta0} = \left[\nu' - \tilde{\nu}'\tan t - \frac{\theta^*(\xi)}{4n}dt(\xi)\tan t\right]g_{\alpha\beta} - i\left[\nu'\tan t + \tilde{\nu}' - \frac{\theta^*(\xi)}{4n}dt(\xi)\right]g_{\alpha\beta};$$

$\mathcal{F}_6$: $M \in h\mathcal{R}_2^{\perp} \oplus h\mathcal{R}_1 \oplus w_1 \oplus w_2 \oplus w_3$,

$$R_{\alpha\beta\gamma\delta} = 2\left[\nu' - i\tilde{\nu}'\right](g_{\beta\gamma}g_{\alpha\delta} - g_{\alpha\gamma}g_{\beta\delta}) + \frac{e^{2it}}{4}\begin{vmatrix} (\mathcal{L}_\xi g)_{\beta\gamma} & (\mathcal{L}_\xi g)_{\alpha\gamma} \\ (\mathcal{L}_\xi g)_{\beta\delta} & (\mathcal{L}_\xi g)_{\alpha\delta} \end{vmatrix},$$
$$R_{\alpha\bar{\beta}\gamma\bar{\delta}} = \frac{1}{4}(\mathcal{L}_\xi g)_{\alpha\gamma}(\mathcal{L}_\xi g)_{\bar{\beta}\bar{\delta}},$$
$$R_{0\alpha\beta0} = \left[\nu' - \tilde{\nu}'\tan t\right]g_{\alpha\beta} - i\left[\nu'\tan t + \tilde{\nu}'\right]g_{\alpha\beta} + i\frac{e^{it}}{4\cos t}dt(\xi)(\mathcal{L}_\xi g)_{\alpha\beta};$$

$\mathcal{F}_{11}$: $M \in \omega_1 \oplus \omega_2 \oplus w\mathcal{R}$,

$$R_{\alpha\beta\gamma\delta} = 2\left[\nu' - i\tilde{\nu}'\right](g_{\beta\gamma}g_{\alpha\delta} - g_{\alpha\gamma}g_{\beta\delta}),$$
$$R_{0\alpha\beta0} = \left[\nu' - \tilde{\nu}'\tan t\right]g_{\alpha\beta} - i\left[\nu'\tan t + \tilde{\nu}'\right]g_{\alpha\beta} + e^{2it}\omega_\alpha\omega_\beta,$$
$$R_{0\bar{\alpha}\beta0} = \omega_{\bar{\alpha}}\omega_\beta.$$

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